VOLUME XLVII NUMBER 1

# THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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FEB 27 1918

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## JANUARY 1918

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THE
# ASTROPHYSICAL JOURNAL
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VOLUME XLVII JANUARY 1918 NUMBER 1

## PHOTOGRAPHY OF THE SOLAR SPECTRUM FROM 6800 A TO 9600 A

By W. F. MEGGERS

Nearly thirty years have passed since Rowland photographed the solar spectrum from its limit in the ultra-violet at 3000 A into the visible red at 7300 A. Many attempts have been made to photograph the spectrum of the sun through the red and out into the adjacent infra-red, but thus far no results have been obtained which are comparable with those in the visible and ultra-violet regions.

The pioneer work in "Mapping the Least Refrangible End of the Solar Spectrum" was described in 1880 by Captain W. de W. Abney,[1] who prepared a silver-bromide emulsion which was sensitive to long waves. With this emulsion the prismatic spectrum of the sun was photographed to about 12000 A, and the diffraction spectrum to 10750 A. The dispersion of glass prisms was too small to show any detail in the spectrum, and less than 200 Fraunhofer lines were shown by the grating spectrum from 7600 A to 9825 A. In 1885 better photographs were obtained, and 590 Fraunhofer lines were recorded between the wave-length limits 7146 A and 9867 A.[2] The gratings were small and the photographic emulsion was

[1] *Philosophical Transactions*, **171**, 653, 1880.

[2] W. de W. Abney, *ibid.*, **177**, 457, 1886.

probably very coarse-grained; hence the definition was poor. The wave-lengths were determined only roughly, making exact coincidences with wave-lengths in spectra of known chemical elements impossible. W. Ritz[1] performed some experiments to improve Abney's emulsion, but the production of such red-sensitive emulsions appears to have been too difficult or uncertain to yield any further results in spectrum photography.

In 1891 George Higgs[2] prepared bisulphite compounds of alizarin blue and cerulin as photographic sensitizers, and succeeded in making an excellent map of the solar spectrum[3] out to 8300 A by the use of photographic plates stained with alizarin blue S. With a narrow slit and long exposures, results in the extreme red end of the spectrum were obtained which possessed most of the detail and definition usually so characteristic of the violet end. No measurements of wave-length were given except for the absorption bands of oxygen A, B, and a.[4]

In 1906 Millochau,[5] at Meudon Observatory, recorded some of the infra-red solar spectrum on photographic plates dyed with green malachite and "solarized" before use. Approximate wave-lengths of only 106 Fraunhofer lines between the limits 8025 A and 9325 A were given.

Up to the present time, therefore, no complete or accurate determinations of wave-lengths corresponding to Fraunhofer lines in the infra-red spectrum have existed, and there have been scarcely any reliable measurements in spectra of the chemical elements in this same region; identifications of absorption and emission lines have therefore been few and uncertain.

During the past year the Bureau of Standards[6] has had considerable success in photographing the red and adjacent infra-red regions of the spectra of chemical elements by means of ordinary photographic plates stained with dicyanin. The process of

[1] *Comptes Rendus*, **143**, 167, 1906.

[2] *Proceedings of the Royal Society*, **49**, 345, 1891.

[3] *Astrophysical Journal*, **7**, 86, 1898.

[4] *Proceedings of the Royal Society*, **54**, 200, 1894.

[5] *Comptes Rendus*, **144**, 725, 1907.

[6] *Bulletin of the Bureau of Standards*, **14**, 372, 1917.

sensitizing the plates was practically that recommended by Dr. Keivin Burns[1] in 1913. About 4 cc of a stock solution of one part dicyanin to one thousand parts of alcohol were added to 50 cc distilled water, 50 cc ethyl alcohol, and 5 cc of strong ammonia. Rapid photographic plates like Seed 27 or Graflex were immersed in such a bath from three to five minutes, rinsed in alcohol, and dried rapidly. Plates treated in this manner were found to be quite sensitive to wave-lengths between 6000 A and 9000 A. Such plates have been used with a six-inch concave grating, having 7500 lines per inch and 21 feet radius, to photograph the spectra of more than forty of the chemical elements. Exposures of thirty minutes recorded waves longer than 9000 A, and exposures of six to eight hours' duration have registered lines beyond 11000 A with arcs of the ferrous metals.

This success with artificial sources suggested an attempt to photograph the infra-red solar spectrum on plates stained with dicyanin. The work was undertaken at the Johns Hopkins University last April and May. For most of this work a five-inch plane grating with 20,000 lines per inch was used, with lenses of 18 feet (548.6 cm) focal length. This spectrograph gave a linear dispersion on the plate of 2 A per mm in the spectrum of the first order. A seven-inch grating with 15,000 lines per inch was also used. The apparatus was mounted on the fourth floor of the same laboratory, in the heart of Baltimore, where Rowland's work was done. In this laboratory disturbances are, no doubt, more frequent, vigorous, and troublesome now than they were in Rowland's time. Vibrations caused by passing street cars, railway trains, and heavy street traffic made it impossible to get satisfactory exposures of more than twenty minutes' duration. In spite of bad weather and hazy and smoky skies, these exposures sufficed to record the solar spectrum with good definition out to 9600 A. Under more favorable conditions this spectrum would have been photographed to still longer wave-lengths. The sensitiveness of dicyanin-stained plates to these long waves is demonstrated by the fact that the long wave limit of the visible spectrum (7600 A) was recorded by exposures of less than one minute. The region 8600 A required

[1] *Journal de physique* (5), **3**, 457, 1913.

only four or five minutes and 9600 A twenty minutes of exposure. Some of these photographs are reproduced in Plates I, II, and III, which show the solar spectrum from 6860 A to 9600 A.[1] Between the limits of wave-length 6800 A and 9600 A the original photographs show nearly 2000 Fraunhofer lines. Comparisons with Rowland's measurements in the region 6800 A to 7330 A show that some of the fine detail is lost by using coarse-grained emulsions for stained plates. By using fine-grained photographic plates and giving from five to ten times as much exposure, all of Rowland's faint lines could probably be registered by means of the dicyanin sensitizer.

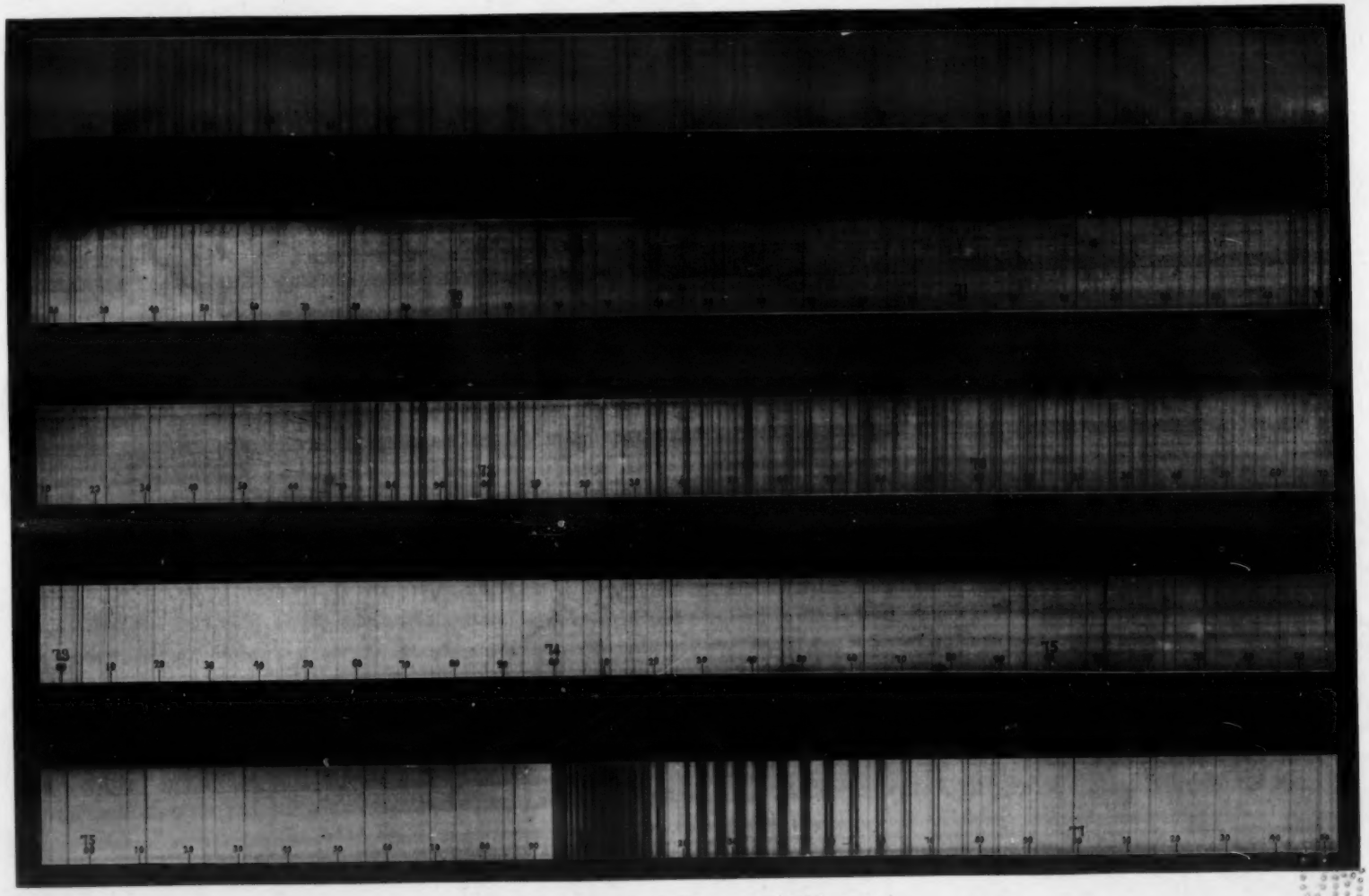
Practically all of the 225 wave-lengths between 6800 A and 9600 A which have been photographed in the spectrum of the iron arc were easily identified with absorption lines in this region of the solar spectrum. These wave-lengths were therefore used as standards for the measurement of wave-lengths corresponding to the remainder of the absorption lines shown by these stained plates. Thus far nearly 400 of these solar wave-lengths have been identified with those of emission lines in the red and infra-red arc spectra of eighteen of the chemical elements. The spectra of eleven other elements have been measured in this spectral region, but none of their lines have been found among solar absorption lines, although some of these elements, notably the rare gases, have their strongest lines in the red and infra-red. Perhaps these rare gases should be looked for in the chromospheric spectrum of the sun. The table shows the number of Fraunhofer lines which have been measured in the region of long waves, and also indicates the number of lines which have thus far been identified with emission lines in the spectra of chemical elements.

In addition to the oxygen bands, many of the absorption lines in the long-wave region of the solar spectrum are, without doubt, of terrestrial origin. This is especially true of the lines near 8230 A and of the bands in the region 9300 A to 9600 A which Langley[2]

[1] It is difficult or impossible to reproduce the faintest lines and close lines which are near the limit of resolution, but at least 75 per cent of the total number of lines seen on the original photographs are shown in these Plates.

[2] *Annals of the Smithsonian Astrophysical Observatory*, 1, 1900.

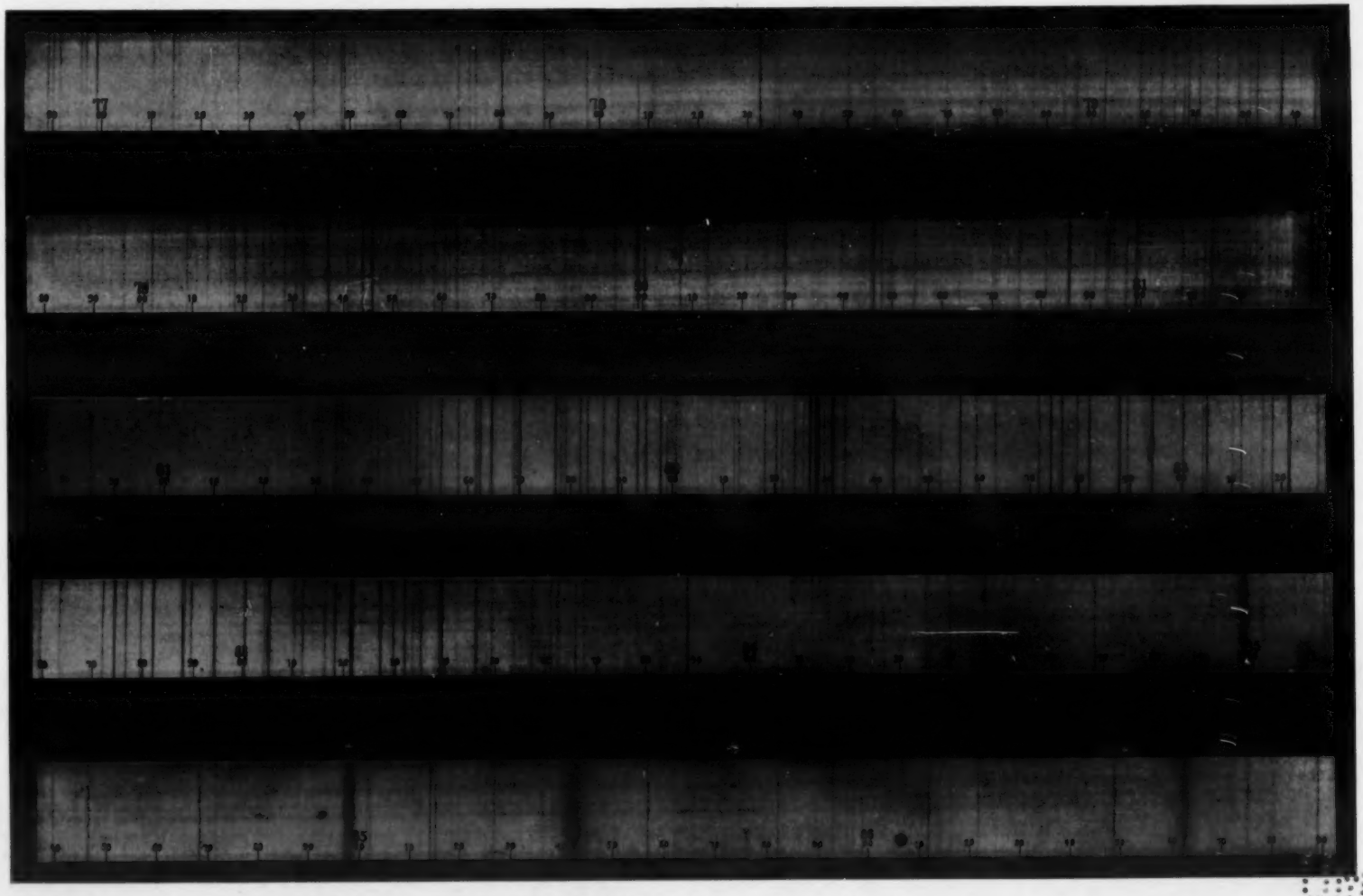
PLATE I



THE INFRA-RED SOLAR SPECTRUM, λ 6860–7750 A

PLATE II

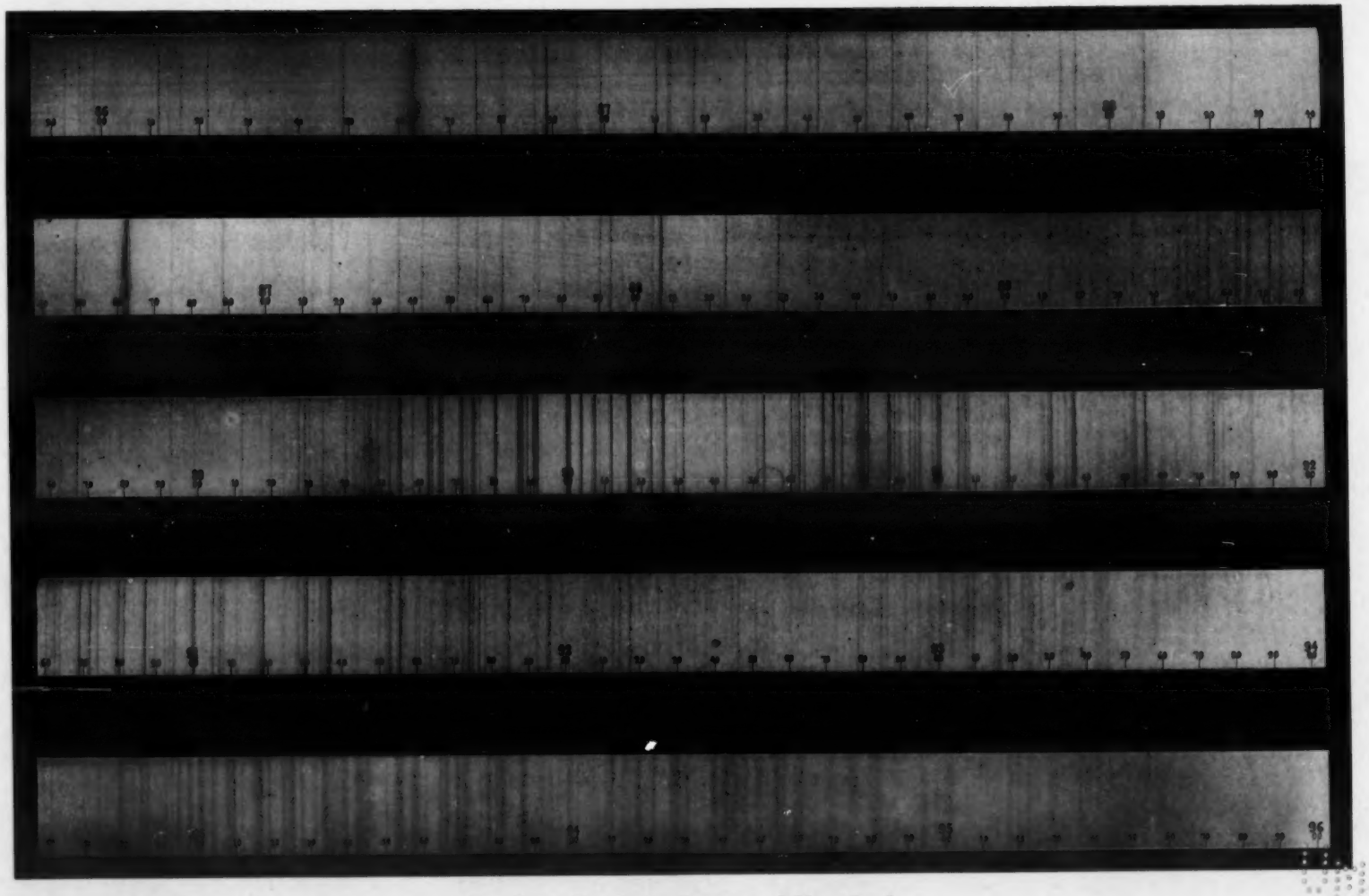
PLATE II



THE INFRA-RED SOLAR SPECTRUM, λ 7700–8690 A

PLATE III

PLATE III



THE INFRA-RED SOLAR SPECTRUM, λ 8600–9600 A

designated $\rho$, $\sigma$, and $\tau$, and believed to be due to water-vapor. There was no opportunity of photographing these when the sun was near the horizon because the walls of the building shielded the heliostat before nine o'clock in the morning and after four o'clock in the afternoon. Photographs taken at noon and three hours later do not show enough difference in line structure in these spectral regions to permit the separation of telluric from solar lines.

NUMBER OF LINES IN THE SOLAR SPECTRUM BETWEEN 6800 A AND 9600 A

| | | | | |
|---|---|---|---|---|
| Iron | 200 | | Barium | 1 |
| Nickel | 63 | | Vanadium | 1 |
| Titanium | 27 | | Selenium | 1 (?) |
| Cobalt | 22 | | Silver | 0 |
| Chromium | 18 | | Lithium | 0 |
| Silicon | 10 | | Rubidium | 0 |
| Manganese | 6 | | Caesium | 0 |
| Calcium | 6 | | Strontium | 0 |
| Tungsten | 3 | (?) | Gold | 0 |
| Magnesium | 2 | | Helium | 0 |
| Sodium | 2 | | Neon | 0 |
| Potassium | 2 | | Argon | 0 |
| Copper | 2 | | Krypton | 0 |
| Molybdenum | 2 | (?) | Xenon | 0 |
| Lanthanum | 2 | (?) | Unidentified | 1600 |

Dr. Schlesinger has recently given the Porter spectrograph[1] at Allegheny Observatory for the continuation of work on the infra-red solar spectrum. This spectrograph was designed for work on the rotation of the sun and is now being used to separate the solar from the telluric lines by means of the displacement suffered by solar lines in consequence of the solar rotation. At the same time an effort is being made to extend photography of the solar spectrum to waves longer than 10000 A.

Publication of the wave-lengths (from 6800 A to 9600 A) determined from the photographs of the solar spectrum made at the Johns Hopkins University will be postponed until further investigations, now in progress at the Bureau of Standards, in this

[1] *Publications of the Allegheny Observatory*, **3**, 99, 1914.

region of the spectra of the chemical elements have been completed. This will make possible the identification of more Fraunhofer lines. In the meantime the work at the Allegheny Observatory will give results for still longer waves, and will separate the solar from the telluric lines.

In conclusion it should be stated that the importance of dicyanin-stained plates for the purpose of photography in the red and infra-red regions of the spectrum cannot be overemphasized. Their convenience and efficiency bring an invisible long-wave spectral region as large as the entire visible spectrum within the reach of simple photography. Their sensitiveness to the long waves recommends their use for the extension of chromospheric and spot spectra as well as for the sun's photospheric spectrum.

BUREAU OF STANDARDS
WASHINGTON, D.C.
November 9, 1917

# A DETERMINATION OF THE SOLAR MOTION AND THE STREAM-MOTION FROM RADIAL VELOCITIES AND ABSOLUTE MAGNITUDES OF STARS OF LATE SPECTRAL TYPES[1]

By GUSTAF STRÖMBERG

In a recent memoir[2] by Adams and myself the results of an investigation concerning the relationship of average radial velocity to absolute magnitude have been published and we have shown that the intrinsically faint stars have a higher velocity than those that are intrinsically brighter. The stars were grouped in different shells according to distance from the sun, and no decided variation of velocity with distance could be found. Continuing this investigation, I have confirmed these results, and have further studied the solar motion and stream-motion as functions of absolute magnitude. This study, as well as the preceding, is based wholly on stars of the spectral types F, G, K, and M. For about 700 of these stars Adams has determined spectroscopically the absolute magnitudes and parallaxes.

When I commenced this work a number of the spectroscopic parallaxes were not definitely determined, and for this reason the absolute motions of the stars in space have not been computed, the radial velocities alone having been used. Hence the absolute magnitude has been employed only for the grouping of the stars. In order to increase the material, use has been made of about 600 determinations of radial velocity from the Lick and Mills Observatories. For these stars mean parallaxes have been computed from their proper motions and apparent magnitudes.

## MEAN PARALLAXES

As mentioned in the paper cited, the recent determinations of stellar parallaxes by van Maanen, and of absolute magnitudes by Adams, indicate that stars of very small proper motion have parallaxes considerably larger than would be expected from their proper

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 144.

[2] *Mt. Wilson Contr.*, No. 131; *Astrophysical Journal*, **45**, 5, 1917.

motions, were the mean parallactic formula of Kapteyn employed. This is perhaps due to the fact that a list of apparently bright stars of late types, chosen on account of small proper motion, involves the selection, not only of distant stars of high intrinsic luminosity, but also of nearer stars whose velocity at right angles to the line of sight is nearly equal in direction and magnitude to the component of the sun's velocity in the same plane. Such a selection of stars of very small proper motion had actually been made in forming the list of stars to be observed for absolute luminosity and radial velocity at the Mount Wilson Observatory.

Kapteyn's formula for computing mean parallaxes from proper motion and apparent magnitude is[1]

$$\pi = A\mu^b\epsilon^m, \qquad (1)$$

where $\mu$ is the proper motion and $m$ the apparent magnitude, $A$, $b$, and $\epsilon$ being constants determined from measured parallaxes and from the parallactic motions of the stars. To render the material homogeneous, new constants were determined from 700 spectroscopically determined parallaxes. It was then found that the equation (1) gave a quite unsatisfactory representation of the smaller parallax. It is to be observed that hitherto the constants for the stars of later type have been based mainly on stars of large parallaxes, the formula for stars of small parallax being merely an extrapolation. Kapteyn's formula, however, is given a theoretical basis by Schwarzschild[2] on the assumption of a constant luminosity-curve and velocity-curve for all stars at all distances. Now if the velocity is a function of absolute magnitude, the velocity-curves and luminosity-curves are dependent on one another; and since among the distant stars we observe only those of high luminosity, both curves are dependent on the distance. There is further a strong tendency, especially among the K and M stars, to a separation of the absolute magnitudes into two groups, one of very high luminosity, the "giants," and one of very low luminosity, the "dwarfs."

[1] *Groningen Publications*, No. 8. Constants redetermined by van Rhijn, *Mt. Wilson Contr.*, No. 110; *Astrophysical Journal*, **43**, 36, 1916.

[2] *Astronomische Nachrichten*, **190**, 361, 1912.

We should also note that the small parallaxes determined spectroscopically can be measured with the same percentage of error as the large parallaxes, provided there are no large systematic errors. In using spectroscopic parallaxes we have the great advantage of securing logarithmic error-distribution within the different groups, as well as in their combination, while in the case of the directly measured parallaxes we have a normal distribution of error in the parallaxes themselves, assuming that we are dealing with a group of stars at the same distance. In Fig. 1 are plotted the values of $\log \pi$ as a function of $\log \mu$ for the spectral types F and G, K and M. The stars are treated separately according as they are near or distant from the sun's apices, and all have been reduced to the apparent magnitude 6.0 on the Harvard system. The points are the means of the logarithms of the parallaxes for different groups of stars of different proper motions. From the figure we see that the straight line

$$\log \pi = \log A + b \log \mu$$

gives a poor representation of the parallaxes of stars of small proper motion. Introducing the constants given by van Rhijn, which differ but slightly from those of Kapteyn, we have

$$\log \pi = -1.078 + 0.695 \log \mu - 0.0482 (m-6). \qquad (2)$$

The equation

$$\log \pi = \log A + b \log (\mu + c) \qquad (3)$$

gives a decidedly better representation of the observations. The constant $b$, which gives the direction of the asymptote for large proper motion, is very difficult of determination, the stars of large motion being too few in number for the purpose. Without affecting the accuracy appreciably, however, we may put $b = 1$, an error in $b$ affecting only the parallaxes of stars of very large proper motion.

Table I gives the values of the constants in the equation

$$\log \pi = \log A' + \log (\mu + c) + (m-6) \log \epsilon \qquad (4)$$

together with their mean errors. The mean error of $\log \pi$ is denoted by $r$, and the angular distance from the sun's apex by $d$.

From the comparison with the observations in Fig. 1 we see that there is a break in the continuity for the stars of types K and M between parallaxes 0".3 and 0".08 (log $\pi = -1.6$ and $-1.1$), corresponding to the division of the stars into giants and dwarfs.

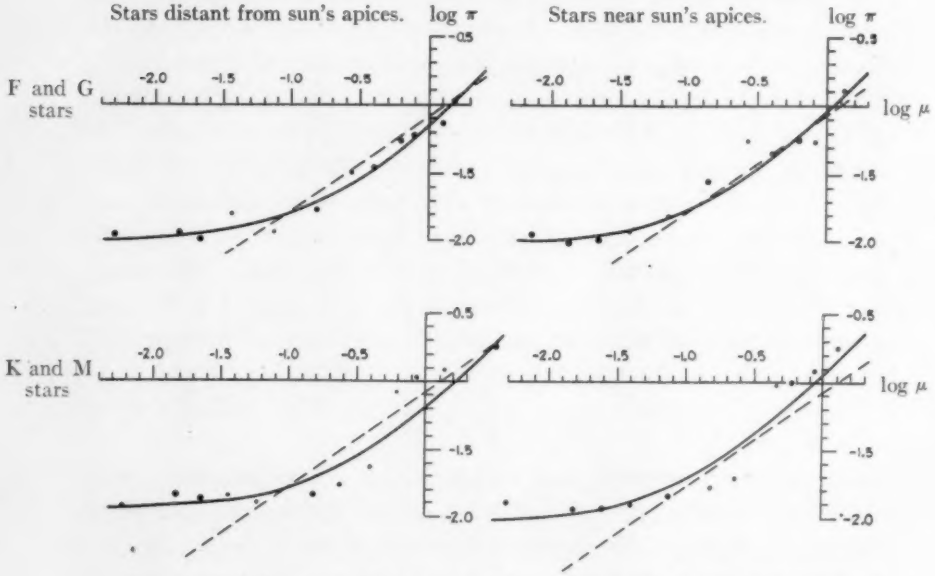



FIG. 1.—Relation between log $\pi$ and log $\mu$. The points represent means of log $\pi$, spectroscopically determined, for groups of stars, all parallaxes having been reduced to the sixth magnitude. The continuous curves represent

$$\log \pi = \log A + \log (\mu + c);$$

the broken lines represent

$$\log \pi = \log A + b \log \mu.$$

TABLE I

CONSTANTS FOR MEAN PARALLAX FORMULA

| $d$ | log $A'$ | $c$ | log $\epsilon$ | $r$ | No. |
|---|---|---|---|---|---|
| | F and G Stars | | | | |
| 60°–120° | −1.208±0.049 | +0".157±0".027 | −0.110±0.015 | ±0.073 | 244 |
| 0°–60°, 120°–180° | −1.081±0.065 | +0.109±0.024 | −0.087±0.040 | ±0.093 | 202 |
| | K and M Stars | | | | |
| 60°–120° | −1.253±0.101 | +0".200±0".049 | −0.101±0.020 | ±0.15 | 117 |
| 0°–60°, 120°–180° | −0.965±0.086 | +0.084±0.033 | −0.115±0.018 | ±0.15 | 135 |

Since there are scarcely any intermediate stars, I have not thought it necessary to compute separate constants for the stars of these two classes.

The difference in size of the points in Fig. 1 represents the difference in weights, which is based directly upon the number of stars in the different groups.

The values of log $\epsilon$ are probably not representative of all stars, since the faint stars are selected mainly on account of large proper motion and thus have parallaxes that are too large.

For values of $\mu$ smaller than about 0".03, the parallax is almost independent of proper motion, and we find

$$\log \pi = \log A' + \log c + (m-6) \log \epsilon. \qquad (5)$$

## SYSTEMATIC ERRORS OF THE PARALLAXES

In a paper by Kapteyn and Adams[1] it was suggested that in case the frequency function of the velocities were different from that given by Maxwell's law, its character would affect seriously the interpretation of the relationship found between proper motion and radial velocity. In order to eliminate this effect, Adams and I have used a formula for mean parallax which is independent of the nature of the frequency function of the velocity components, and based only on the assumption that for a group of stars scattered over the whole sky this function is the same in all directions. A brief account of the derivation of the formula and of the final equations used was included in our paper. A more detailed and complete statement is given here.

The star's velocity in space corrected for the sun's motion is projected on three axes, one coinciding with the radial velocity; of the other two, at right angles to the line of sight, one is directed toward the sun's apex and the other is at right angles thereto. The three components of the star's velocity in space, corrected for the sun's motion, are therefore

$$V' = V + V_0 \cos d; \quad V_v = \frac{kv}{\pi} + V_0 \sin d; \quad V_r = \frac{k\tau}{\pi}, \qquad (6)$$

[1] *Mount Wilson Communications*, No. 1; *Proceedings of the National Academy of Sciences*, 1, 14, 1915.

in which $V$ is the star's radial velocity, $V_0$ the sun's velocity, $d$ the angle between the star and the sun's apex, $\nu$ and $\tau$ the components of the proper motion in the directions mentioned, and $k$ a constant equal to 4.738 km/sec. Assuming the frequency function F to be the same in these three directions, we have

$$F(V') = F(V_\nu) = F(V_\tau).$$

In a group of stars the logarithms of the parallaxes are distributed more nearly in accordance with a normal frequency-curve than are the parallaxes themselves. In forming the mean parallax we therefore deal with the logarithms and use the geometrical instead of the arithmetical mean, and write accordingly,

$$\text{Geometrical Mean Parallax} = \underline{\pi} = \sqrt[n]{\Pi(\pi)},$$

whence

$$\log \underline{\pi} = \frac{1}{n}\Sigma \log \pi = \overline{\log \pi},$$

in which $n$ is the number of individual parallaxes. A further advantage of this method of procedure is that the ratios in equation (6) are thus transformed into differences from which we can form directly $\overline{\log \pi}$ or $\log \underline{\pi}$, whatever the range in the parallax.

We have, therefore,

$$F_1 (\log V') = F_1 (\log V_\tau) \text{ or } \overline{\log V'} = \overline{\log V_\tau}$$

where $V'$ is to be taken regardless of sign. We then find

$$\overline{\log \pi} = \log k + \overline{\log \tau} - \overline{\log V'}$$

or

$$\underline{\pi} = \frac{k\tau}{\underline{V'}}, \tag{7}$$

a bar above the symbol denoting the arithmetical mean, a bar below the symbol the geometrical mean.

For the $\nu$ component we have

$$F(V') = F(V_\nu) = F\left(\frac{k\nu}{\pi} + V_0 \sin d\right).$$

Hence,

$$F_1(V' - V_0 \sin d) = F_1\left(\frac{k\nu}{\pi}\right)$$

and

$$F_2[\log (V'-V_0 \sin d)]=F_2\left(\log \frac{k\nu}{\pi}\right)$$

or, finally,

$$\overline{\log \pi}=\log k+\overline{\log \nu}-\overline{\log (V'-V_0 \sin d)} \qquad (8)$$

or

$$\underline{\pi}=\frac{k\nu}{\overline{V'-V_0 \sin d}}.$$

Since $F(V')$ can be assumed to be equal to $F(-V')$ we also find

$$F(-V')=F\left(\frac{k\nu}{\pi}+V_0 \sin d\right)$$

or

$$F_1[\log (-V'-V_0 \sin d)]=F_1[\log (V'+V_0 \sin d)]=F_1\left(\log\frac{k\nu}{\pi}\right).$$

Hence,

$$\overline{\log \pi}=\log k+\overline{\log \nu}-\overline{\log (V'+V_0 \sin d)} \qquad (8a)$$

or

$$\underline{\pi}=\frac{k\nu}{\overline{V'+V_0 \sin d}}.$$

Taking the mean of (8) and (8*a*) we find

$$\overline{\log \pi}=\log k+\overline{\log \nu}-\tfrac{1}{2}[\overline{\log (V'+V_0 \sin d)}+\overline{\log (V'-V_0 \sin d)}]. \quad (9)$$

For a large group of stars scattered over the whole sky there can be only a small effect of preferential motion, and the assumption of the equality of the frequency function in the three components probably is nearly fulfilled. Furthermore, among the stars of later type the *K*-term, which can be interpreted either as a systematic error in the determination of radial velocity or as a general dilatation or contraction of the stellar system, appears to be very small. In the paper by Adams and myself already cited, we found for parallaxes larger than 0".017 a good agreement between the values computed from the formulae (7) and (8) and the parallaxes actually employed, the latter being in part spectroscopic determinations and in part computed from formula (5). For smaller parallaxes, however, equations (7) and (8) gave smaller values than those actually employed.

Formulae (7) and (9) have now been applied to a large number of parallaxes determined spectroscopically by Adams in order to find the systematic errors in his values. Stars of proper motion smaller than 0".020 annually have been excluded. Table II con-

TABLE II

SYSTEMATIC ERROR OF SPECTROSCOPIC PARALLAXES

| $M$ | No. | $\bar{m}$ | $\bar{M}_A$ | $\underline{\pi}_A$ | $\underline{\pi}_\tau$ | $\underline{\pi}_\nu$ | $\frac{1}{2}(\underline{\pi}_\tau+\underline{\pi}_\nu)$ | $\underline{V'}$ | $\bar{M}_1$ | $\frac{O-C}{\bar{M}_A-\bar{M}_1}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | A8–F7 Stars | | | | | | | | | |
| | | | | | | | | km | | |
| ≦3.9 | 32 | 4.81 | +2.54 | 0".035 | 0".034 | 0".043 | 0".038 | 10.3 | +2.71 | −0.17 |
| ≧4.0 | 33 | 5.89 | +4.78 | 0.060 | 0.041 | 0.057 | 0.049 | 17.8 | +4.34 | +0.44 |
| | F8–G7 Stars | | | | | | | | | |
| ≦3.9 | 81 | 5.07 | +1.49 | 0.019 | 0.024 | 0.020 | 0.022 | 11.0 | +1.79 | −0.30 |
| ≧4.0 | 104 | 6.61 | +5.34 | 0.056 | 0.049 | 0.079 | 0.064 | 23.9 | +5.64 | −0.30 |
| | G8–K4 Stars | | | | | | | | | |
| ≦1.9 | 89 | 5.04 | +1.00 | 0.0156 | 0.0161 | 0.0176 | 0.0168 | 10.3 | +1.16 | −0.16 |
| ≧2.0 | 91 | 6.87 | +5.26 | 0.0477 | 0.0421 | 0.0460 | 0.0441 | 18.9 | +5.09 | +0.17 |

tains the result of the comparison. $\bar{M}_A$ is the mean absolute magnitude as determined by Adams, $\bar{m}$ the mean apparent magnitude, and $\underline{\pi}_A = 10^{0.2(M-\bar{m})-1}$ the geometrical mean of the spectroscopic parallaxes; $\underline{\pi}_\tau$ and $\underline{\pi}_\nu$ have been computed by equations (7) and (9). $\underline{V'}$ is the geometrical mean of the radial velocities and $\bar{M}_1$ the absolute magnitude computed from

$$\bar{M}_1 = \bar{m} + 5 + 5 \log \pi, \tag{10}$$

where

$$\pi = \tfrac{1}{2}(\underline{\pi}_\nu + \underline{\pi}_\tau).$$

Formulae (7) and (9) for the mean parallax appear to possess some advantages over those commonly used when the range in proper motion is large. It is not necessary that the variation in $\pi$ be very small, which is an essential condition both in Campbell's formula $\bar{\pi} = k\bar{\tau}/\bar{V}'$ and in the formula for computing mean parallaxes from the parallactic motion, namely,

$$\Sigma V_\nu = 0,$$

or

$$\bar{\pi} = -\frac{k\overline{v \sin d}}{V_0 \overline{\sin^2 d}}.$$

In order to secure this result, the stars usually are divided into groups of the same proper motion and apparent magnitude, but we then often obtain groups which are too small to furnish values of $\pi$ accurate enough for deriving absolute magnitudes. Further if the geometrical mean of $\pi$ is used, we can compute directly from the mean apparent magnitude the mean absolute magnitude, whatever the range in $\pi$.

The largest difference in absolute magnitude in Table II is only 0.4, corresponding to an error in the parallax of $\frac{1}{5}$ of its value. We may therefore assume that the systematic errors in the spectroscopic parallaxes are comparatively small.

### AVERAGE RADIAL VELOCITY AND ABSOLUTE MAGNITUDE

In the paper by Adams and myself it was stated that a very pronounced correlation exists between absolute magnitude and average radial velocity. In that investigation types F and G were combined as well as types K and M. I have now treated types F, G, and K separately and have found as before that no appreciable effect of distance can be detected by studying separately stars at nearly the same distance. In addition, I have computed the so-called $K$-term, i.e., the algebraic mean of the radial velocities corrected for the sun's motion. With reversed sign this term can be regarded as the systematic correction to the measured radial velocities, on the assumption that the algebraic sum of the radial velocities corrected for the sun's motion is equal to zero.

The results are in Table III. $\overline{M}$ is the mean absolute magnitude, $\overline{m}$ the mean apparent magnitude, $\underline{\pi}$ the geometrical mean of the parallaxes, $\underline{V'}$ the geometrical mean of the radial velocities corrected for the sun's motion, $\theta$ the average radial velocity regardless of sign, and $K$ the algebraic mean of the radial velocities. The radial velocities are here corrected for the sun's motion, the constants

$$A = 270^\circ, \qquad D = +30^\circ, \qquad V = 20.0 \text{ km}$$

being assumed. The ratio of the number of stars within galactic latitudes $+30°$ and $-30°$ to the whole number of stars in the group is denoted by $c$, which is, accordingly, a measure of the galactic condensation. Were the stars uniformly distributed over the sky the value of $c$ would be 0.5.

TABLE III

RADIAL VELOCITY AND ABSOLUTE MAGNITUDE

| $M$ | No. | $\overline{M}$ | $\overline{m}$ | $\pi$ | $\underline{V'}$ | $c$ | No. | $\theta$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|
| | F Stars | | | | | | | | |
| | | | | | km | | | km | km |
| ≦0.9 | 56 | 0.24 | 4.16 | 0".016 | 7.0 | 0.86 | 56 | 10.8 | +0.8 |
| 1.0–1.9 | 73 | 1.54 | 4.83 | .022 | 9.7 | .55 | 73 | 14.4 | +0.8 |
| 2.0–2.9 | 66 | 2.42 | 5.32 | .026 | 8.6 | .64 | 65 | 12.9 | −1.5 |
| ≧3.0 | 104 | 4.28 | 5.70 | .052 | 15.3 | .45 | 92 | 16.3 | −1.0 |
| | 299 | 2.44 | 5.12 | 0.029 | 10.4 | 0.59 | 286 | 14.0 | −0.3 |
| | G Stars | | | | | | | | |
| ≦0.9 | 152 | 0.31 | 4.91 | .012 | 7.8 | .66 | 149 | 11.3 | +1.3 |
| 1.0–1.9 | 112 | 1.38 | 5.76 | .013 | 10.0 | .50 | 108 | 13.1 | +0.2 |
| 2.0–3.9 | 58 | 2.74 | 5.60 | .027 | 14.5 | .50 | 51 | 17.2 | −2.7 |
| 4.0–5.9 | 73 | 4.97 | 6.49 | .050 | 15.3 | .42 | 59 | 15.7 | −2.3 |
| ≧6.0 | 36 | 6.90 | 7.36 | .081 | 20.2 | .39 | 30 | 21.2 | −4.9 |
| | 431 | 2.25 | 5.70 | 0.020 | 11.0 | 0.53 | 397 | 14.1 | −0.5 |
| | K Stars | | | | | | | | |
| ≦0.9 | 124 | 0.52 | 4.18 | .018 | 8.5 | .72 | 124 | 13.5 | +1.6 |
| 1.0–1.9 | 251 | 1.42 | 4.84 | .021 | 11.6 | .54 | 251 | 16.5 | +1.8 |
| 2.0–4.9 | 108 | 2.59 | 5.08 | .032 | 14.7 | .53 | 107 | 21.0 | +6.1 |
| ≧5.0 | 78 | 7.10 | 7.45 | .085 | 17.4 | .46 | 78 | 27.6 | −7.9 |
| | 561 | 2.23 | 5.11 | 0.027 | 12.0 | 0.56 | 560 | 18.3 | +1.2 |

Both the geometrical means, $\underline{V'}$, and the arithmetical means, $\theta$, of the radial velocities show a decided increase with decreasing brightness. There is, however, for all three types an apparent deviation from a linear relationship between radial velocity and absolute magnitude. If we plot the numbers, we find a break in the straight line at absolute magnitude 2 for F stars, at 3 or 4 for G stars, and between 4 and 6 for K stars. The same break occurs in the $K$-term, the absolutely bright stars having a positive $K$-term

and the absolutely faint a negative $K$-term. This break in the continuity is probably related to the division of stars into the giant and dwarf classes.

If we assume the mean velocity to be a function of the mass rather than of the absolute brightness, the more massive stars having smaller velocity than the less massive, we find that the ratio of luminosity to mass is larger for the giants than for the dwarfs. This is in harmony with Adams' results that the low-temperature lines are strengthened in the spectra of the dwarf stars.

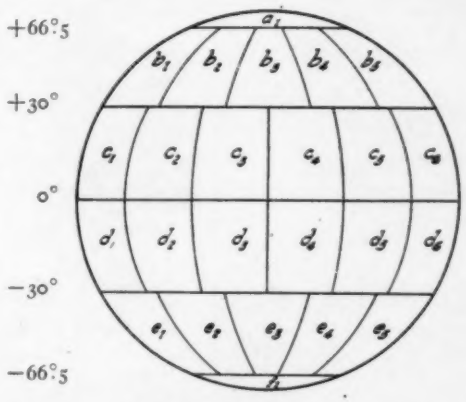



FIG. 2.—Distribution of forty-eight standard regions in the sky.

From Table III we find further that the very luminous stars show a much larger galactic concentration than the absolutely faint stars, the latter seeming rather to avoid the galaxy.

## SOLAR MOTION AND ABSOLUTE MAGNITUDE

In order to find whether any decided relationship exists between the solar motion and the absolute magnitude, I have grouped the stars according to absolute magnitude, taking types F and G together. Only the radial velocities have been used in this investigation. Adopting a method of Charlier,[1] I divided the sky into 48 regions of equal area; but instead of using the equator as a plane of symmetry, I have grouped the stars in areas symmetrical to the galactic plane. This was done mainly to simplify the study of the stream-motion. Fig. 2 shows the location and designation of the areas.

Regions $c_1$ to $c_{12}$ and $d_1$ to $d_{12}$ are situated along the galactic equator between latitudes $+30°$ and $-30°$, the origin of longitude

[1] *Meddelanden från Lunds Observatorium*, Ser. II, No. 8, 1912.

being the intersection of the galactic and celestial equators. The regions $b_1$ to $b_{10}$ and $e_1$ to $e_{10}$ are situated between latitudes $\pm 31°$ and $\pm 66°$. Regions $a$ and $f$ are in latitudes higher than $\pm 67°$.

The centers of gravity of regions $a_1$ to $c_{12}$ are given in Table IV. The centers for areas $d$, $e$, and $f$ have the same longitudes, but galactic latitudes $-14°.48$, $-45°.1$, and $-80°$, respectively.

TABLE IV

CENTERS OF GRAVITY OF STANDARD REGIONS

| Region | Gal. Long. | Gal. Lat. | Region | Gal. Long. | Gal. Lat. |
|---|---|---|---|---|---|
| $a_1$ | 90° | +80° | $c_1$ | 15° | +14°.48 |
| $a_2$ | 270 | +80 | $c_2$ | 45 | +14.48 |
| $b_1$ | 18 | +45.1 | $c_3$ | 75 | +14.48 |
| $b_2$ | 54 | +45.1 | $c_4$ | 105 | +14.48 |
| $b_3$ | 90 | +45.1 | $c_5$ | 135 | +14.48 |
| $b_4$ | 126 | +45.1 | $c_6$ | 165 | +14.48 |
| $b_5$ | 162 | +45.1 | $c_7$ | 195 | +14.48 |
| $b_6$ | 198 | +45.1 | $c_8$ | 225 | +14.48 |
| $b_7$ | 234 | +45.1 | $c_9$ | 255 | +14.48 |
| $b_8$ | 270 | +45.1 | $c_{10}$ | 285 | +14.48 |
| $b_9$ | 306 | +45.1 | $c_{11}$ | 315 | +14.48 |
| $b_{10}$ | 342 | +45.1 | $c_{12}$ | 345 | +14.48 |

The pole of the galactic plane is assumed to have the co-ordinates[1] $\alpha = 190°.6$, $\delta = +27°.2$ (1900).

The equations for finding equatorial from galactic co-ordinates are then

$$\left.\begin{aligned} \cos\alpha\cos\delta &= +0.1846x + 0.4494y - 0.8740z \\ \sin\alpha\cos\delta &= -0.9828x + 0.0844y - 0.1642z \\ \sin\delta &= \quad\quad + 0.8893y + 0.4573z \end{aligned}\right\} \qquad (11)$$

where

$$x = \cos l\cos b, \qquad y = \sin l\cos b, \qquad z = \sin b. \qquad (12)$$

The use of the galactic co-ordinates instead of equatorial is of no advantage in the computation of the solar apex, but is very convenient in the study of the stream-motion.

The equation of condition for the determination of the sun's motion from radial velocities is

$$\gamma_1 X_0 + \gamma_2 Y_0 + \gamma_3 Z_0 + K = V, \qquad (13)$$

[1] *Groningen Publications*, No. 18, 1908.

which is to be computed for each region separately, $\gamma_1$, $\gamma_2$, and $\gamma_3$ being the direction-cosines of the center of gravity of the regions, $\bar{V}$ the algebraic mean of the observed radial velocities, $K$ the $K$-term, and $X_0$, $Y_0$, and $Z_0$ the components of the motion of the centroid of the group of stars relative to the sun. Further,

$$X_0 = -V_0 \cos B_0 \cos L_0, \quad Y_0 = -V_0 \cos B_0 \sin L_0, \quad Z_0 = -V_0 \sin B_0 \quad (14)$$

where $L_0$ and $B_0$ are the galactic co-ordinates of the sun's apex and $V_0$ the sun's velocity.

The different regions have been given weights proportional to the number of stars in each. The results, with mean errors appended, are in Table V; $\theta$ is the average radial velocity.

TABLE V

SUN'S APEX AND VELOCITY

| $M$ | No. | $\bar{M}$ | $\bar{m}$ | $A$ | $D$ | $V_0$ | $K$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| | F and G Stars | | | | | | | |
| | | | | | | km | km | km |
| ≦0.9 | 211 | 0.31 | 4.68 | 251°.6 ± 6°.6 | +22°.7 ± 6°.1 | 19.4 ±1.8 | + 0.2 ±1.8 | 11.4 |
| 1.0–1.9 | 177 | 1.44 | 5.42 | 267.5 ± 11.0 | +36.3 ± 12.4 | 16.6 ±2.4 | + 0.0 ±1.5 | 14.6 |
| 2.0–3.9 | 167 | 2.76 | 5.27 | 271.6 ± 9.0 | +34.6 ± 10.0 | 22.0 ±2.8 | − 2.1 ±1.7 | 16.2 |
| ≧4.0 | 170 | 5.29 | 6.41 | 279.7 ± 9.5 | (+ 7.4 ± 9.9) | (26.5 ±4.1) | − 1.9 ±2.7 | 23.9 |
| | 725 | 2.32 | 5.40 | 268.1 ± 4.1 | +25.2 ± 4.9 | 19.94 ± 1.38 | − 0.69 ± 0.85 | 17.15 |
| | K Stars | | | | | | | |
| ≦0.9 | 122 | 0.54 | 4.22 | 282°.4 ± 9°.2 | +32.8 ± 8.8 | 23.6 ±2.6 | + 2.0 ±1.6 | 13.6 |
| 1.0–1.9 | 245 | 1.41 | 4.86 | 268.6 ± 8.3 | +36.5 ± 9.2 | 20.5 ±2.4 | + 1.1 ±1.4 | 16.6 |
| 2.0–4.9 | 99 | 2.58 | 5.12 | 282.1 ± 9.9 | +16.1 ± 9.4 | 26.4 ±4.6 | + 5.8 ±2.4 | 18.1 |
| ≧5.0 | 79 | 7.07 | 7.41 | (285.9 ± 28.9) | (−13.0 ± 34.4) | (15.3 ±6.6) | (−11.1 ±4.9 | 26.5 |
| | 545 | 2.25 | 5.13 | 277.5 ± 5.5 | +32.2 ± 5.8 | 22.23 ± 1.73 | + 1.05 ± 1.00 | 18.46 |
| | Giant M Stars | | | | | | | |
| Bright | 135 | 1.5 | 4.98 | 264°.8 ± 7°.4 | +26.3 ± 8.2 | 27.0 ±3.3 | + 1.25 ± 1.86 | 16.9 |

There is an indication that the absolutely faint stars give a smaller declination of the sun's apex than the brighter stars, which is in harmony with the result that stars of large proper motion give a smaller declination of the apex than stars of small motion. This is usually explained on the two-drift theory: Drift I having the

greater velocity relative to the sun, the stars belonging to it have on an average greater proper motions than those of Drift II. The relation to absolutely faint stars is partly the result of a selection of stars of large proper motion. The results for the faint stars, however, are very uncertain, as these have been observed mainly in the northern sky only (Mount Wilson Observatory).

Putting $K=0$, the discrepancy in the declination of the sun's apex is much diminished. The results corresponding to this assumption are in Table V*a*.

TABLE V*a*

SUN'S APEX AND VELOCITY—*K*-TERM = 0

| $\overline{M}$ | $A$ | $D$ | $V_\odot$ | $\theta$ |
|---|---|---|---|---|
| | | F and G Stars | | |
| | | | km | km |
| 0.31 | 251°.4 | +22.5 | 19.4 | 11.3 |
| 1.44 | 267.5 | +36.3 | 16.6 | 14.6 |
| 2.76 | 272.1 | +36.4 | 22.8 | 16.3 |
| 5.29 | 279.6 | +10.9 | 27.1 | 23.9 |
| 2.32 | 268.3 | +26.1 | 20.13 | 17.15 |
| | | K Stars | | |
| 0.54 | 279.6 | +33.8 | 24.0 | 13.7 |
| 1.41 | 268.1 | +37.2 | 20.4 | 16.6 |
| 2.58 | 284.5 | +20.1 | 26.0 | 18.6 |
| 7.07 | 289.0 | +26.5 | 22.1 | 26.2 |
| 2.25 | 277.0 | +32.5 | 22.16 | 18.5 |
| | | Giant M Stars | | |
| 1.5 | 264.2 | +26.1 | 26.8 | 16.9 |

From all later-type stars (excluding dwarf M stars) we find for the solar motion

| | |
|---|---|
| $A_\odot = 270°.9 \pm 3°.28$ | $K = 0$ (assumed) |
| $D_\odot = +29.2 \pm 3.45$ | $A_\odot = 270°.7$ |
| $V_\odot = 21.48 \pm 1.02$ km/sec | $D_\odot = +29.0$ |
| $\theta = 17.71$ km/sec | $V_\odot = 21.42$ km/sec |
| $K = +0.36 \pm 0.60$ km/sec | $\theta = 17.71$ km/sec |

No. of stars 1405

The values of $\theta$ are found a posteriori from the least-squares solution. Their increase with the decreasing brightness is well shown. There is no pronounced relationship between absolute magnitude and solar motion, except the decrease of declination for the intrinsically faint stars, which probably is partly an effect of different proportions of stars belonging to the two drifts. It is to be observed that these faint stars cannot be compared with apparently faint stars, which according to Dyson and others give a high declination for the sun's apex. The latter are mainly very distant stars, while the dwarfs are stars at very small distances.

## PREFERENTIAL MOTIONS

The preferential motion of the stars used in this investigation has been determined from the radial velocities alone. Instead of using the dispersion—that is, the square root of the sum of the squares of radial velocities corrected for the solar motion divided by the number of stars—I have used the average radial velocity as a measure of the mobility in the line of sight. Squaring the velocities exaggerates the effect of a few high velocities, and even if we exclude velocities larger than a certain limit, there is often too great a number of high velocities near this limit. (Cf. Eddington, *Stellar Movements*, p. 147.)

The grouping of stars according to absolute brightness is very useful for the study of preferential motion, since the different groups are then more homogeneous as regards their mean velocities and only a very few stars of high velocity have to be excluded. Since the reductions for each star were computed before the solar motion was determined, I assumed for the latter $A = 270°$, $D = +30°$, and $V_0 = 20.0$ km. These differ slightly from the values just found for the later-type stars, but the effect of the difference is quite inappreciable. No systematic correction has been applied to the measured radial velocities ($K$-term) since the values found in Table V have rather large errors, and are themselves probably too small to affect the result appreciably. The average velocity corrected for the sun's motion was computed for each region, the corrections for the sun's motion being determined separately for each star, and the

co-ordinates of the center of the region were used for computing the velocity surface.

I have made use neither of the ellipsoidal theory nor of the method for computing the two drifts, but have employed a more general method, which enables me to take into account the difference in mean velocity at opposite points of the sky. We know that the sun is somewhat north of the real galactic plane, and, further, that the B-type stars and the apparently faint stars in general are not uniformly distributed in galactic longitude. If in accordance with Turner's suggestion we assume the stream-motion to be a gravitational effect, we cannot expect a priori the velocity surface to be symmetrical with respect to the sun, but may anticipate it to be related in some way to the eccentric position of the sun in the stellar system. Since the mean distance of the late-type stars is of the same order of magnitude as the distance to the center of the stellar system, we may expect asymmetrical (odd) terms in the analytical expression of the velocity surface. Such asymmetrical terms would also exist if the velocity of the stream-motion were different in different regions of space, which has been suggested by Kapteyn (*Mount Wilson Annual Report*, 1916, p. 255).

We desire to express the average radial velocity as a continuous function of the longitude and latitude of the different regions. We start from the following rational integral function of the direction-cosines:[1]

$$\begin{aligned} r = {} & a_0 + b_1 x + b_2 y + b_3 z + c_1 x^2 + c_2 y^2 + c_3 z^2 + c_4 xy + c_5 xz \\ & + c_6 yz + d_1 x^3 + d_2 y^3 + d_3 z^3 + d_4 x^2 y + d_5 x y^2 + d_6 x^2 z \\ & + d_7 x z^2 + d_8 y^2 z + d_9 y z^2 + d_{10} xyz + \ldots . \end{aligned} \tag{15}$$

in which $r$ is the radius vector and equal to the average radial velocity $\theta$, and $x$, $y$, and $z$ are the direction-cosines of a point on the surface as defined by equation (12). We then have

$$x^2 + y^2 + z^2 = 1. \tag{16}$$

The intersection of this surface with the $XY$-plane has the equation

$$r = a_0 + b_1 x + b_2 y + c_1 x^2 + c_2 y^2 + c_4 xy + d_1 x^3 + d_2 y^3 + d_4 x_2 y + d_5 x y^2 + \ldots . \tag{17}$$

[1] Such a surface including terms of second order only was used by Eddington, *Monthly Notices*, **75**, 521, 1915, as representing approximately an ellipsoidal velocity-distribution.

Introducing the longitudes we find

$$r=\alpha_0+\beta_1 \cos l+\beta_2 \sin l+\gamma_1 \cos 2l+ \gamma_2 \sin 2l+ \ldots . \qquad (18)$$

whence we see that (17) can be represented by harmonic terms of the same order as the degree of equation (15). This property holds for every intersection of the surface with a plane through the origin, for the form of equation (15) is unaltered if we change the direction of the axes of the system of co-ordinates. The arguments of the harmonic terms are thus multiples of longitudes reckoned in the intersection plane. Taking into account this property, we find that the number of independent coefficients in the equation of the surface is $(n+1)^2$ where $n$ is the degree of the equation. In equation (15) some of the coefficients are thus superfluous, a fact depending on the circumstance that $x$, $y$, and $z$ are connected by equation (16). These we shall eliminate as follows.

It is convenient to define the coefficients in the expression for the surface in such a way that the terms in $x$, $y$, and $z$ represent deviations from the mean radius vector. We therefore introduce the condition $r=a_0$, $r$ being the mean of all the radii vectores to the surface. The integral over the sphere for all other terms must be equal to zero. This is the case for each of the terms as they stand, except those involving $x^2$, $y^2$, and $z^2$. The condition that the integrals for the sum of these terms may also be zero is found as follows: Since $x$, $y$, and $z$ are direction-cosines, the definite integrals of $x^2$, $y^2$, and $z^2$ over the sphere are equal. Denoting the value of these integrals by $\sigma$, the required condition is

$$(c_1+c_2+c_3)\sigma=0,$$

whence

$$c_3=-c_1-c_2.$$

To eliminate the remaining superfluous coefficients we write the identities

$$\left.\begin{aligned} \alpha x^3+\alpha xy^2+\alpha xz^2-\alpha x=0 \\ \beta x^2y+\beta y^3+\beta yz^2-\beta y=0 \\ \gamma x^2z+\gamma y^2z+\gamma z^3-\gamma z=0 \end{aligned}\right\} \qquad (19)$$

where $\alpha$, $\beta$, and $\gamma$ are quite arbitrary. Assuming

$$d_7+\alpha=d_8+\gamma=d_9+\beta=0$$

and adding equations (15) and (19) and simplifying, we find

$$r=\bar{r}+B_1x+B_2y+B_3z+C_1x^2+C_2y^2-(C_1+C_2)z^2+C_3xy+C_4xz \\ +C_5yz+D_1x^3+D_2y^3+D_3z^3+D_4x^2y+D_5xy^2+D_6x^2z+D_7xyz, \tag{20}$$

which contains no superfluous coefficients.

Introducing now longitudes and latitudes defined by equation (12), we find the following trigonometrical expression for the surface:

$$r=r+\beta_1\cos b\cos l+\beta_2\cos b\sin l+\beta_3\sin b+\gamma_1\cos^2 b\cos 2l \\ +\gamma_2\cos^2 b\sin 2l+\gamma_3(2/3-\cos^2 b)+\gamma_4\sin 2b\cos l \\ +\gamma_5\sin 2b\sin l+\delta_1,\cos^3 b\cos 3l+\delta_2\cos^3 b\sin 3l \\ +\delta_3\cos^3 b\cos l+\delta_4\cos^3 b\sin l+\delta_5\sin^3 b \\ +\delta_6\cos^2 b\sin b\cos 2l+\delta_7\cos^2 b\sin b\sin 2l. \tag{21}$$

Equation (21) is the expression for the surface to terms of the third order inclusive. For application I have changed the form of (21) somewhat in order to separate the terms more easily and to secure nearly the same variation in the different trigonometrical combinations, so that the coefficients may indicate nearly the real effect of the different terms.

The form used is:

$$\theta=a_0+a_1\cos b\cos l+a_2\cos b\sin l+a_3\sin b+a_4\cos^2 b\cos 2l \\ +a_5\cos^2 b\sin 2l+a_6(2-3\cos^2 b)+a_7\sin 2b\cos l \\ +a_8\sin 2b\sin l+a_9\cos^3 b\cos 3l+a_{10}\cos^3 b\sin 3l \\ +a_{11}\cos 3b\cos l+a_{12}\cos 3b\sin l+a_{13}\sin 3b \\ +a_{14}\,8/3\cos^2 b\sin b\cos 2l+a_{15}\,8/3\cos^2 b\sin b\sin 2l. \tag{22}$$

The terms with coefficients $a_1$ to $a_3$ are of the first order, $a_4$ to $a_8$ of the second order, and $a_9$ to $a_{15}$ of the third order: i.e., they have 1, 2, and 3 maxima and minima, respectively. Each term represents a deviation from the sphere $\theta=\bar{\theta}=a_0$ but does not affect the mean value of $\bar{\theta}=a_0$ for the whole sphere.

Turning to Tables III and V, we find that average velocity is more closely related to absolute luminosity than to spectral type. For this reason and in order to obtain groups large enough for studying the variation in average velocity for different regions in the sky, the stars were divided into three groups having different radial velocities, each including stars of types F, G, and K. The first group contains the stars intrinsically brightest with absolute mag-

nitudes equal to or less than 1.9 for F and G stars, and equal to or less than 0.9 for the K stars. The average radial velocity for this group varies from 11.4 to 14.6 km (see Table V). In the second group of stars, of absolute magnitudes 2.0−3.9 for F and G stars and 1.0−4.9 for the K stars, the average radial velocity ranges from 16.2 to 18.1 km. In the last group all stars fainter than these limits were taken together, with average velocity ranging from 23.9 to 26.5 km. This group also includes 11 dwarf M stars with an average radial velocity of about 30 km. These groups are more homogeneous with regard to velocity and probably even to mass than groups based solely on spectral types. Further data are given in Table VI.

TABLE VI

GROUPS FOR STUDY OF PREFERENTIAL MOTION

| Group | No. | $\overline{M}$ | $\overline{m}$ | $\overline{\pi}$ | $\theta$ |
|---|---|---|---|---|---|
| | | | | | km |
| I Very bright..... | 509 | 0.76 | 4.83 | 0".015 | 13.11 |
| II Bright..... | 513 | 2.08 | 5.09 | 0.025 | 17.13 |
| III Faint..... | 260 | 6.05 | 6.82 | 0.070 | 25.88 |

The average velocity for the stars of these groups in each of the standard regions, corrected for the sun's motion, is given in Table VII. The problem is the representation of these data by the surface defined by equation (22).

TERMS OF EVEN DEGREE

Since the terms of odd degree are equal and opposite in sign for opposite points in the sky, we may eliminate them by using the means for opposite regions. We can thus derive independently the terms of even degree. The corresponding equation,[1] representing radial velocities, is

$$\theta = a_0 + a_4 \cos^2 b \cos 2l + a_5 \cos^2 b \sin 2l + a_6 (2 - 3\cos^2 b) + a_7 \sin 2b \cos l + a_8 \sin 2b \sin l.$$

In combining opposite regions the average radial velocity was computed by giving weights proportional to the number of stars; but for the determination of the constants of the surface, the 24 means

[1] This expression was used by Hough and Halm for computing the solar motion on the assumption of two opposite streams (*Monthly Notices*, **70**, 94, 1909).

TABLE VII

VELOCITIES FOR STANDARD REGIONS

| REGION | I | | II | | III | | REGION | I | | II | | III | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\theta$ | No. | $\theta$ | No. | $\theta$ | No. | | $\theta$ | No. | $\theta$ | No. | $\theta$ | No. |
| | km | | km | | km | | | km | | km | | km | |
| $a_1$ | ..... | 0 | 8.3 | 7 | 35.3 | 11 | $f_1$ | 11.5 | 10 | 8.4 | 6 | 15.2 | 3 |
| $a_2$ | 5.9 | 4 | 18.2 | 13 | 25.8 | 3 | $f_2$ | 16.0 | 7 | 23.6 | 9 | 5.4 | 2 |
| | 5.9 | 4 | 14.7 | 20 | 33.3 | 14 | | 13.3 | 17 | 17.5 | 15 | 11.3 | 5 |
| $b_1$ | 11.8 | 12 | 18.4 | 17 | 31.4 | 14 | $e_1$ | 12.2 | 14 | 11.2 | 9 | 39.7 | 4 |
| $b_2$ | 8.2 | 5 | 17.6 | 10 | 16.2 | 8 | $e_2$ | 12.2 | 6 | 15.3 | 12 | 27.9 | 9 |
| $b_3$ | 13.0 | 12 | 20.3 | 6 | (20.3) | 1 | $e_3$ | 13.0 | 16 | 15.7 | 11 | 25.2 | 13 |
| $b_4$ | 11.6 | 12 | 16.9 | 9 | 32.2 | 13 | $e_4$ | 13.9 | 9 | 21.3 | 7 | 21.9 | 9 |
| $b_5$ | 14.8 | 17 | 16.2 | 10 | 26.4 | 13 | $e_5$ | 15.8 | 4 | 23.6 | 8 | 23.7 | 3 |
| $b_6$ | 16.1 | 6 | 16.3 | 10 | (34.5) | 2 | $e_6$ | 14.0 | 4 | 8.8 | 15 | (36.6) | 1 |
| $b_7$ | 11.6 | 5 | 21.9 | 8 | 8.6 | 5 | $e_7$ | 9.4 | 7 | 19.6 | 14 | 25.2 | 3 |
| $b_8$ | 5.9 | 5 | 13.6 | 5 | 4.1 | 7 | $e_8$ | 8.6 | 5 | 18.1 | 16 | (18.1) | 1 |
| $b_9$ | 8.0 | 4 | 17.7 | 11 | 31.0 | 8 | $e_9$ | 11.1 | 8 | 21.4 | 8 | (30.8) | 5 |
| $b_{10}$ | 14.0 | 3 | 17.0 | 14 | 35.8 | 8 | $e_{10}$ | 12.5 | 9 | 17.6 | 9 | (36.3) | 1 |
| | 12.20 | 81 | 17.63 | 100 | 25.7 | 79 | | 12.25 | 82 | 16.68 | 109 | 27.1 | 49 |
| $c_1$ | 10.8 | 15 | 22.1 | 12 | 20.2 | 9 | $d_1$ | 12.9 | 10 | 9.2 | 8 | 30.3 | 11 |
| $c_2$ | 11.4 | 24 | 14.5 | 11 | 18.2 | 8 | $d_2$ | 14.3 | 26 | 19.1 | 18 | 23.8 | 8 |
| $c_3$ | 14.8 | 12 | 15.3 | 21 | 22.9 | 7 | $d_3$ | 14.0 | 18 | 14.0 | 12 | 19.0 | 11 |
| $c_4$ | 5.0 | 9 | 9.6 | 12 | 14.5 | 6 | $d_4$ | 16.0 | 18 | 19.9 | 21 | 26.7 | 6 |
| $c_5$ | 10.4 | 13 | 16.2 | 8 | 39.0 | 9 | $d_5$ | 11.7 | 16 | 18.9 | 7 | 36.9 | 3 |
| $c_6$ | 17.2 | 18 | 14.8 | 8 | 19.0 | 6 | $d_6$ | 18.0 | 12 | 20.2 | 8 | 32.0 | 8 |
| $c_7$ | 18.2 | 11 | 20.0 | 7 | 27.5 | 4 | $d_7$ | 18.9 | 12 | 35.9 | 7 | ..... | 0 |
| $c_8$ | 14.4 | 8 | 15.4 | 9 | ( 0.8) | 1 | $d_8$ | 11.2 | 14 | 14.3 | 10 | ( 4.7) | 1 |
| $c_9$ | 9.2 | 12 | 9.7 | 10 | ..... | 0 | $d_9$ | 13.3 | 19 | 11.6 | 12 | ( 0.7) | 1 |
| $c_{10}$ | 9.1 | 4 | 16.3 | 14 | (68.0) | 1 | $d_{10}$ | 8.3 | 10 | 16.3 | 13 | (25.7) | 1 |
| $c_{11}$ | 18.6 | 9 | 15.7 | 12 | ( 6.0) | 2 | $d_{11}$ | 16.6 | 12 | 26.3 | 13 | ..... | 0 |
| $c_{12}$ | 16.6 | 8 | 17.6 | 8 | 29.7 | 8 | $d_{12}$ | 13.7 | 15 | 27.8 | 8 | 43.0 | 2 |
| | 13.07 | 143 | 15.47 | 132 | 24.1 | 61 | | 14.09 | 182 | 19.03 | 137 | 26.5 | 52 |

thus obtained were given equal weight. The following values of the coefficients and their mean errors were found from a least-squares solution.

| | Coefficient | I | II | III |
|---|---|---|---|---|
| | | km | km | km |
| | $a_0$ | 12.97±0.42 | 17.29±0.58 | 25.46±1.33 |
| $\cos^2 b \cos 2l$ | $a_4$ | + 2.54±0.75 | + 2.62±1.04 | + 5.13±2.37 |
| $\cos^2 b \sin 2l$ | $a_5$ | − 0.51±0.75 | − 1.81±1.04 | − 5.68±2.37 |
| $2-3\cos^2 b$ | $a_6$ | − 0.58±0.46 | − 0.42±0.63 | + 0.80±1.44 |
| $\sin 2b \cos l$ | $a_7$ | − 0.09±0.76 | + 0.90±1.05 | − 1.76±2.39 |
| $\sin 2b \sin l$ | $a_8$ | − 0.86±0.75 | + 0.77±1.03 | − 0.30±2.35 |

The last three coefficients have rather large errors as compared with the coefficients themselves, and hence they might be excluded in computing the axes. Since all the terms, however, are of equal geometrical significance, we can secure a little more accuracy by taking all into account. The very small term $-0.09 \sin 2b \cos l$, however, has been omitted.

Introducing direction-cosines, we find

$$\begin{aligned}
&\text{I} \quad \theta = 12.97 + 3.12x^2 - 1.96y^2 - 1.16z^2 - 1.02xy - 0.18xz - 1.72yz \\
&\text{II} \quad \theta = 17.29 + 3.04x^2 - 2.20y^2 - 0.85z^2 - 3.62xy + 1.80xz + 1.54yz \\
&\text{III} \quad \theta = 25.46 + 4.33x^2 - 5.93y^2 + 1.60z^2 - 11.36xy - 3.52xz - 0.60yz
\end{aligned}$$

In forming the maxima and minima of these expressions for the determination of the position of the axes of the surfaces, we must take into account the condition (16). We therefore write equal to zero the partial derivatives of the function

$$\theta - \lambda (x^2 + y^2 + z^2 - 1),$$

in which $\lambda$ is an arbitrary coefficient.

The direction-cosines $x$, $y$, and $z$ of the axes of the surface are therefore given by the equations

$$\text{Group I} \ldots \begin{cases} (+3.12-\lambda)x - 0.51y - 0.09z = 0 \\ -0.51x + (-1.96-\lambda)y - 0.86z = 0 \\ -0.09x - 0.86y + (-1.16-\lambda)z = 0 \end{cases}$$

$$\text{Group II} \ldots \begin{cases} (+3.04-\lambda)x - 1.81y + 0.90z = 0 \\ -1.81x + (-2.20-\lambda)y + 0.77z = 0 \\ +0.90x + 0.77y + (-0.85-\lambda)z = 0 \end{cases}$$

$$\text{Group III} \ldots \begin{cases} (+4.33-\lambda)x - 11.36y - 3.52z = 0 \\ -11.36x + (-5.93-\lambda)y - 0.60z = 0 \\ -3.52x - 0.60y + (1.60-\lambda)z = 0 \end{cases}$$

The discriminating cubics of these equations are

$$\begin{aligned}
&\text{I} \quad \lambda^3 - 8.469\lambda - 5.025 = 0 \\
&\text{II} \quad \lambda^3 - 12.081\lambda - 5.940 = 0 \\
&\text{III} \quad \lambda^3 - 170.04\lambda + 223.64 = 0
\end{aligned}$$

The roots of the cubics are

| | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
|---|---|---|---|
| I | + 3.17 | −0.62 | − 2.55 |
| II | + 3.70 | −0.50 | − 3.20 |
| III | +12.32 | +1.33 | −13.65 |

The axes of the surfaces thus computed are in Table VIII.

The maximum axis, which can be assumed to be identical with the axis of preferential motion in the ellipsoidal and the two-drift theory, lies in all cases near the galactic equator. The intermediate

TABLE VIII

VERTICES OF MOTION. TERMS OF EVEN DEGREE

| Group | Axis | $\theta$ | $l$ | $b$ | $\alpha$ | $\delta$ |
|---|---|---|---|---|---|---|
| | | km | | | | |
| I | Maximum | 16.14 | 174°.3 | + 0°.1 | 98°.0 | + 5°.1 |
| | Minimum | 10.42 | 84.3 | +31.9 | 236.0 | +83.2 |
| | Third direction | 12.35 | 264.4 | +58.1 | 187.7 | − 4.5 |
| II | Maximum | 20.98 | 164.0 | − 8.0 | 85.9 | +10.3 |
| | Minimum | 14.09 | 70.4 | −23.6 | 348.4 | +35.7 |
| | Third direction | 16.79 | 91.8 | +64.9 | 189.4 | +52.3 |
| III | Maximum | 32.38 | 148.5 | +14.1 | 99.6 | +34.1 |
| | Minimum | 17.32 | 56.2 | + 9.1 | 303.7 | +53.4 |
| | Third direction | 25.58 | 294.2 | +73.1 | 197.6 | +11.7 |

axis lies in all cases nearest to the galactic pole.

For comparison I give here a list of other determinations of the principal vertex mainly from Eddington, *Stellar Movements*, p. 124:

| | | $\alpha$ | $\delta$ |
|---|---|---|---|
| Distant stars: | Kapteyn | 91° | +13° |
| | Rudolph | 96 | + 7 |
| | Hough and Halm | 90 | + 8 |
| | Eddington | 95 | + 3 |
| | Schwarzschild | 93 | + 6 |
| | Eddington | 94 | +12 |
| | *Hough and Halm | 88 | +27 |
| | *Gyllenberg[1] | 84 | + 5 |
| | Raymond[2] | 92 | + 4 |
| | *Present Papers (I and II) | 92 | + 8 |
| Nearer stars: | Charlier | 103 | +19 |
| | Dyson | 88 | +21 |
| | Beljawsky | 86 | +24 |
| | Raymond[2] | 92 | +35 |
| | *Present Paper (III) | 100 | +34 |

(The asterisk indicates that the determination is based on radial velocities.)

[1] *Meddelanden från Lunds Observatorium*, Ser. II, No. 13, 1915.

[2] *Astronomical Journal*, 30, 191, 1917.

There is here some indication that the nearer stars yield a higher declination for the principal vertex. The same result is found in the present investigation, the declination of the vertex having values $+5^\circ.1$, $+10^\circ.3$, and $+34^\circ.1$; and it is even more marked in the galactic longitudes $174^\circ.3$, $164^\circ.0$, $148^\circ.5$.

### ASYMMETRICAL TERMS

In order to find if there are systematic differences in the average radial velocities in opposite regions, I have computed the terms of the first and third orders in the expression for the surface given by equation (22). One-half of the difference of the average velocities for opposite regions represents their effect. The number of stars in some regions of the third groups being too small, I have discussed in this way only stars of the first and second groups.

The expression used for the asymmetrical terms is

$$\Delta\theta = a_1 \cos b \cos l + a_2 \cos b \sin l + a_9 \cos^3 b \cos 3l + a_{10} \cos^3 b \sin 3l + a_{11} \cos 3b \cos l + a_{12} \cos 3b \sin l + a_{13} \sin 3b + a_{14}\, 8/3 \cos^2 b \sin b \cos 2l + a_{15}\, 8/3 \cos^2 b \sin b \sin 2l.$$

The term $a_3 \sin b$ is omitted, the stars near the pole of the galaxy being too few to yield a good determination. In the second group I have replaced the term $a_{13} \sin 3b$ by the fifth-order term $a_{16} \sin 4b \cos b$, which proved to be very large.

Assigning equal weights to the different regions, a least-squares solution gave the following values for the coefficients:

| | | I | II |
|---|---|---|---|
| | | km | km |
| $\cos b \cos l$ | $a_1$ | $-1.21 \pm 0.45$ | $-0.47 \pm 0.91$ |
| $\cos b \sin l$ | $a_2$ | $+0.84 \pm 0.45$ | $-0.35 \pm 0.91$ |
| $\cos^3 b \cos 3l$ | $a_9$ | $-1.94 \pm 0.52$ | $-1.87 \pm 1.07$ |
| $\cos^3 b \sin 3l$ | $a_{10}$ | $-2.15 \pm 0.52$ | $-2.47 \pm 1.07$ |
| $\cos 3b \cos l$ | $a_{11}$ | $+0.74 \pm 0.53$ | $+0.25 \pm 1.09$ |
| $\cos 3b \sin l$ | $a_{12}$ | $-1.34 \pm 0.53$ | $-0.29 \pm 1.09$ |
| $\sin 3b$ | $a_{13}$ | $-0.66 \pm 0.38$ | ..... |
| $8/3 \cos^2 b \sin b \cos 2l$ | $a_{14}$ | $+0.98 \pm 0.47$ | $+0.14 \pm 0.97$ |
| $8/3 \cos^2 b \sin b \sin 2l$ | $a_{15}$ | $+0.32 \pm 0.47$ | $+2.73 \pm 0.97$ |
| $\sin 4b \cos b$ | $a_{16}$ | ..... | $-2.35 \pm 0.89$ |

For the second group a new solution was made putting $a_1 = a_2 = a_{11} = a_{12} = a_{14} = 0$. The values were not changed, but the mean

errors of the four remaining terms were reduced to $\pm 0.94$, 0.94, 0.85, and 0.78, respectively.

The complete expressions for the surfaces used in computing the maxima and minima are thus

$$\begin{aligned} \text{I}\quad \theta = {} & 12.97 - 1.21 \cos b \cos l + 0.84 \cos b \sin l + 2.54 \cos^2 b \cos 2l \\ & - 0.51 \cos^2 b \sin 2l - 0.86 \sin 2b \sin l - 1.94 \cos^3 b \cos 3l \\ & - 2.15 \cos^3 b \sin 3l + 0.74 \cos 3b \cos l - 1.34 \cos 3b \sin l \\ & + 0.98 \times 8/3 \cos^2 b \sin b \cos 2l + 0.32 \times 8/3 \cos^2 b \sin b \sin 2l \\ & - 0.58 (2 - 3 \cos^2 b) - 0.66 \sin 3b. \end{aligned}$$

$$\begin{aligned} \text{II}\quad \theta = {} & 17.29 + 2.62 \cos^2 b \cos 2l - 1.81 \cos^2 b \sin 2l + 0.90 \sin 2b \cos l \\ & + 0.77 \sin 2b \sin l - 1.87 \cos^3 b \cos 3l - 2.47 \cos^3 b \sin 3l \\ & + 2.73 \times 8/3 \cos^2 b \sin b \sin 2l - 0.42 (2 - 3 \cos^2 b) \\ & - 2.35 \sin 4b \cos b. \end{aligned}$$

In Figs. 3 and 4 are shown the intersections of these surfaces with the galactic equator. The radii vectores of the dots represent the average radial velocity for the squares along the galaxy, i.e.,

TABLE IX

INTERSECTIONS OF VELOCITY SURFACE WITH GALACTIC EQUATOR

| REGION | GAL. LONG. | I | | | | II | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | No. | $\theta_{obs.}$ | $\theta_{comp.}$ | O−C | No. | $\theta_{obs.}$ | $\theta_{comp.}$ | O−C |
| | | | km | km | km | | km | km | km |
| $c_1+d_1$ | 15° | 25 | 11.6 | 12.0 | −0.4 | 20 | 16.9 | 16.0 | +0.9 |
| $c_2+d_2$ | 45 | 50 | 12.9 | 12.2 | +0.7 | 29 | 17.3 | 15.5 | +1.8 |
| $c_3+d_3$ | 75 | 30 | 14.3 | 13.4 | +0.9 | 33 | 14.8 | 17.6 | −2.8 |
| $c_4+d_4$ | 105 | 27 | 12.3 | 11.4 | +0.9 | 33 | 16.1 | 16.8 | −0.7 |
| $c_5+d_5$ | 135 | 29 | 11.1 | 11.1 | 0.0 | 15 | 17.5 | 16.5 | +1.0 |
| $c_6+d_6$ | 165 | 30 | 17.5 | 16.2 | +1.3 | 16 | 17.5 | 20.4 | −2.9 |
| $c_7+d_7$ | 195 | 23 | 18.6 | 19.0 | −0.4 | 14 | 28.0 | 22.1 | +5.9 |
| $c_8+d_8$ | 225 | 22 | 12.3 | 13.9 | −1.6 | 19 | 14.8 | 16.3 | −1.5 |
| $c_9+d_9$ | 255 | 31 | 11.7 | 8.8 | +2.9 | 22 | 10.7 | 11.5 | −0.8 |
| $c_{10}+d_{10}$ | 285 | 14 | 8.5 | 11.8 | −3.3 | 27 | 16.3 | 15.9 | +0.4 |
| $c_{11}+d_{11}$ | 315 | 21 | 17.4 | 17.0 | +0.4 | 25 | 21.2 | 22.6 | −1.4 |
| $c_{12}+d_{12}$ | 345 | 23 | 14.8 | 15.8 | −1.0 | 16 | 22.7 | 21.3 | +1.4 |
| | | 325 | | | | 269 | | | |

$c_1+d_1$, $c_2+d_2$, etc., whose values are in Table IX. The dotted curves are the intersections with the galactic equator of the surfaces containing only terms of second order.

For both groups of stars we find a very marked three-lobed curve of nearly the same general appearance and position in the galactic equator. The three maxima are produced by the large third-order terms $a_9 \cos^3 b \cos 3l$ and $a_{10} \cos^3 b \sin 3l$. The sums of these terms for the two groups are

I $2.90 \cos^3 b \cos 3(l-76°.0)$ II $3.10 \cos^3 b \cos 3(l-77°.6)$

with mean errors in the coefficients of $\pm 0.74$ and $\pm 1.33$, respectively. They are of the same magnitude as the second-order terms which determine the stream-motion, namely,

I $2.59 \cos^2 b \cos 2(l-174°.3)$ II $3.18 \cos^2 b \cos 2(l-162°.7)$

the mean errors here being $\pm 1.06$ and $\pm 1.47$. The evidence for these third-order terms has therefore much the same definiteness as that for the existence of the stream-motion.

The maxima of the surfaces were obtained by assuming approximate values of the longitudes and latitudes of the axes and computing corrections to their co-ordinates with the aid of the formulae

$$\Delta b \frac{\partial^2 \theta}{\partial b^2} + \Delta l \frac{\partial^2 \theta}{\partial b \partial l} + \frac{\partial \theta}{\partial b} = 0$$

$$\Delta b \frac{\partial^2 \theta}{\partial b \partial l} + \Delta l \frac{\partial^2 \theta}{\partial l^2} + \frac{\partial \theta}{\partial l} = 0$$

the derivatives being calculated with approximate value of $b$ and $l$. The two principal maxima of the radius vector for each surface are given in Table X.

TABLE X

CO-ORDINATES OF MAXIMUM RADIAL VELOCITIES

| Group | Maximum $\theta$ | $l$ | $b$ | $\alpha$ | $\delta$ |
|---|---|---|---|---|---|
| | km | | | | |
| I..... | 19.60 | 190° | + 4° | 109° | − 7° |
| | 17.50 | 324 | − 2 | 264 | −33 |
| II..... | 24.23 | 179 | −18 | 84 | − 7 |
| | 26.33 | 322 | −19 | 283 | −41 |

An approximately symmetrical plane perpendicular to the galactic equator was found for the two surfaces in longitudes

I $L = 258°.6 \pm 5°.9$ II $L = 256°.6 \pm 3°.0$

Each of these directions nearly bisects the angle between the two maximal axes of the surfaces. In Figs. 3 and 4 the directions of the planes of symmetry are indicated by arrows.

### VELOCITY SURFACE OF THE DWARF STARS

In order to find the longitudes of the maxima of the average radial velocities, a harmonic analysis was made taking together all stars between galactic latitudes $-66°$ and $+66°$. The values of $\theta$ for the different regions are found in Table XI.

TABLE XI

AVERAGE VELOCITIES—DWARF STARS

| Region | Gal. Long. | No. | $\theta_{obs.}$ | $\theta_{comp.}$ | O−C |
|---|---|---|---|---|---|
| | | | km | km | km |
| $b_1+c_1+c_1+d_1$ | 16°.5 | 38 | 29.3 | 29.6 | −0.3 |
| $b_2+c_2+c_2+d_2$ | 49.5 | 33 | 21.7 | 21.6 | +0.1 |
| $b_3+c_3+c_3+d_3+c_4+d_4$ | 90.0 | 44 | 21.9 | 23.2 | −1.3 |
| $b_4+c_4+c_5+d_5$ | 130.5 | 34 | 31.7 | 28.6 | +3.1 |
| $b_5+c_5+c_6+d_6$ | 163.5 | 30 | 26.2 | 29.7 | −3.5 |
| $b_6+c_6+c_7+d_7$ | 196.5 | 7 | 30.8 | 27.3 | +3.5 |
| $b_7+c_7+c_8+d_8$ | 229.5 | 10 | 12.4 | 15.3 | −2.9 |
| $b_8+c_8+c_9+d_9+c_{10}+d_{10}$ | 270.0 | 11 | 12.8 | 11.7 | +1.1 |
| $b_9+c_9+c_{11}+d_{11}$ | 310.5 | 15 | 27.6 | 27.6 | 0.0 |
| $b_{10}+c_{10}+c_{12}+d_{12}$ | 343.5 | 19 | 34.0 | 33.9 | +0.1 |

The computed average velocities were derived from the expression

$$\theta = 24.35 + 1.78 \cos l + 3.76 \sin l + 6.88 \cos 2l - 4.88 \sin 2l - 0.17 \cos 3l - 2.01 \sin 3l,$$

which is the result of the analysis.

The maximum values of $\theta$ are in longitudes 157° and 340°, and there is further a pronounced minimum in longitude 252°. The longitude of the symmetrical plane is 254°, in close agreement with that found for the more distant stars. The curve is given in Fig. 5.

### TENTATIVE EXPLANATION OF THE PROPERTIES OF THE VELOCITY SURFACE

It is possible that the asymmetrical form of surface representing the average radial velocities $\theta = f(b, l)$ depends upon the eccentric position of the sun in the galactic system of stars. This suggestion

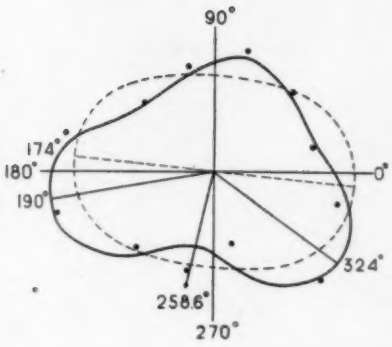



FIG. 3

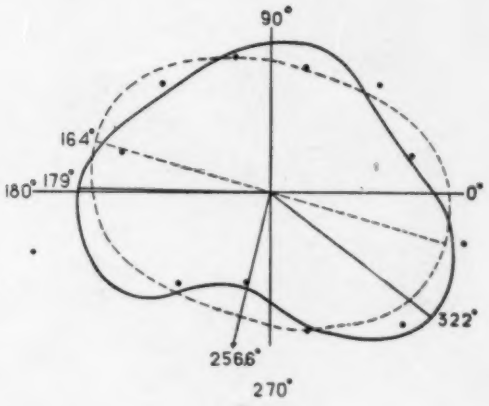



FIG. 4

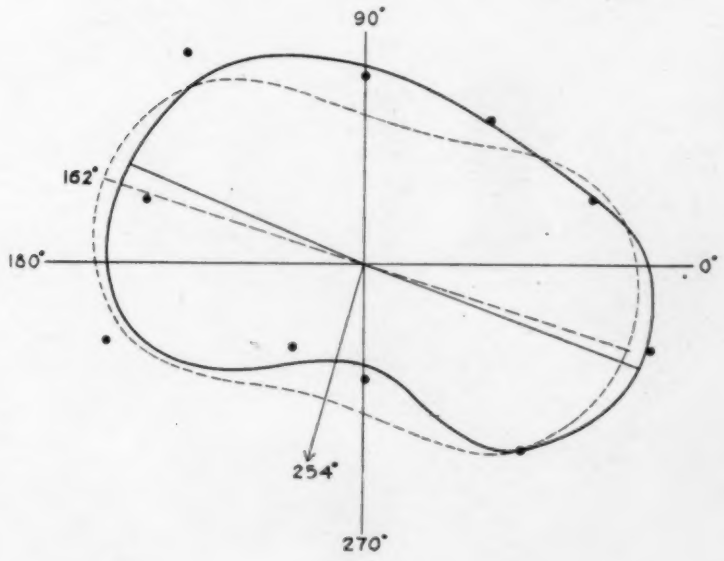



FIG. 5

FIGS. 3, 4, 5.—Intersections between the average radial-velocity surfaces and the galactic equator for the three groups of stars—Fig. 3 for the stars intrinsically brightest and most distant, Fig. 5 for those intrinsically faintest and nearest, and Fig. 4 for the stars intermediate in luminosity and distance. The radii vectores of the points represent the average radial velocity (1 cm=9 km/sec.) in regions adjacent to the galactic equator. The full curves are the intersections when terms to the third order inclusive are taken into account; the dotted curves, when terms of the second order only are used. The projections of the maximal axes of the surfaces are indicated by straight lines; the arrows indicate the longitudes of approximately symmetrical planes perpendicular to the galactic equator.

is supported by the fact, derived by a consideration of the terms depending on galactic latitude alone, that in all groups the average radial velocity is a maximum somewhat south of the galactic equator.

We thus find the following expressions for the variation in latitude alone:

$$\text{I} \quad \theta = 12.97 - 0.58\,(2 - 3\cos^2 b) - 0.66 \sin 3b$$
$$\text{II} \quad \theta = 17.29 - 0.42\,(2 - 3\cos^2 b) - 2.35 \sin 4b \cos b$$
$$\text{III} \quad \theta = 25.46 - 0.37\,(2 - 3\cos^2 b) + 4.95 \sin b - 4.94 \sin 3b$$

the last being derived from a separate analysis of the average velocities in the six different zones. The curve corresponding to Group II is given in Fig. 6.

All these functions have maximum values of $\theta$ for negative values of the latitude. Thus,

$$\text{I} \quad \theta_{\max.} = 13.9 \text{ km} \qquad b = -19^\circ.1$$
$$\text{II} \quad \theta_{\max.} = 19.7 \text{ km} \qquad b = -19.9$$
$$\text{III} \quad \theta_{\max.} = 28.4 \text{ km} \qquad b = -22.8^{1}$$

Since the sun is situated north of the real galactic plane at a distance of about 20 parsecs, the result may possibly indicate a maximum of space velocity in this plane.

The position of the center of the galactic system has recently been determined by Charlier[2] and by Walkey.[3] Charlier gives for the co-ordinates of the center of 800 B-type stars

$$\alpha = 115^\circ.5, \quad \delta = -55^\circ.6; \qquad l = 236^\circ, \quad b = -14^\circ$$
$$\text{Distance} = 18.21 \text{ siriometers} = 88.3 \text{ parsecs.}$$

Walkey finds for 30,736 stars of all types $l = 246^\circ$, and for the B and A stars separately

$$834 \text{ B stars}, \; l = 239^\circ$$
$$10{,}337 \text{ A stars}, \; l = 250^\circ$$

The last value agrees well with the longitude of the planes symmetrical to the velocity surfaces ($258^\circ.6$, $256^\circ.6$, $254^\circ$).[4]

[1] This, however, is not the principal maximum which lies at the galactic north pole.

[2] *Meddelanden från Lunds Observatorium*, Ser. II, No. 14, 1916.

[3] *Monthly Notices*, **74**, 649, 1914.

[4] A new determination by H. Nort of the star-densities as a function of galactic longitude has recently appeared. He finds from the Harvard Map of stars down to eleventh magnitude a largest concentration at longitude 275° (*Recherches Astronomiques de l'observatoire d'Utrecht*, **7**, 1917).

We may assume that the maximum of average radial velocity occurs when the angle between the line of sight and the direction of preferential motion in space is a minimum. The fact that the two maximal axes of the surface are inclined to each other might therefore be interpreted as a result of a preferential motion around the center in both directions. The plane of symmetry of the surface would then contain the center around which the stars are moving. The agreement between the longitudes of the symmetrical planes and the longitude of the center of the galaxy as determined by Charlier and Walkey supports this hypothesis. The increase in the values of the difference in the longitudes of the axes of maximum radial velocity (134°, 143°, and 183°), as we pass from the distant stars to the nearer, is also in harmony with such a suggestion. Further, if the stars are moving around the center of the stellar system, we may expect a minimum of orbital velocity near the center.[1] Such a minimum is perhaps indicated by the exceptionally small values of the radial velocity near longitude 257°.

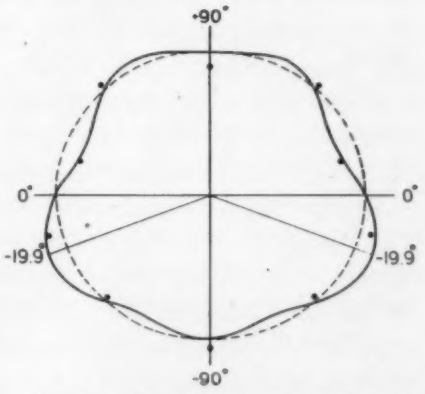



FIG. 6.—Intersection of a plane through the axis of the galaxy with the surface of revolution representing the average radial velocities for stars of Group II when terms depending on galactic latitude alone are included. Dotted curve corresponds to terms of even orders.

That the stream-motion possibly is a motion around the center of the galaxy is further indicated by the fact that the axis of preferential motion is in the galactic plane and nearly perpendicular to the direction toward the center of the stellar system.

The increase in average radial velocity with decreasing brightness is well shown in Figs. 3–5, which are drawn to the same scale. For almost every region when separately considered the radius

[1] Strömgren, "Über Bewegungsformen in Globular Clusters," *Astronomische Nachrichten*, **203**, 17, 1916.

vector increases with decreasing brightness in passing from Group I to Group III.

## SUMMARY OF RESULTS

The radial velocities of stars of spectral types F, G, and K have been studied, the stars being grouped according to absolute luminosity with the aid of Adams' spectroscopic parallaxes. The relationship of proper motion to parallax has been determined, and the systematic errors in these parallaxes have been calculated by assuming the same frequency-distribution of the velocity-components in the line of sight as at right angles to it. The variation with absolute brightness of average radial velocity, of the $K$-term, of galactic concentration, of the solar motion, and of the stream-motion has been studied. The following are the main results.

1. In the formula of Kapteyn for computing mean parallax from proper motion and apparent magnitude it is necessary to add a term $c$ to the proper motion in order to secure satisfactory agreement with the measured parallaxes of the very distant stars. The parallaxes of these stars appear to be almost independent of proper motion.

2. The systematic errors in Adams' spectroscopically determined parallaxes are small, the maximum error being one-fifth of the parallaxes.

3. For stars of types F, G, and K there seems to be a steady increase of average radial velocity with decreasing brightness. For all three types there seems to be a break in the linear relationship, corresponding to the division into giants and dwarfs. If we suppose the relation between velocity and luminosity to be an effect of mass, the giants probably have a higher ratio of luminosity to mass than the dwarfs.

4. The intrinsically bright stars have a higher galactic concentration than the fainter stars.

5. The $K$-term seems to be positive for the brightest stars and negative for the fainter stars.

6. No decided relationship between the constants of the solar motion and absolute brightness could be found. The intrinsically faint stars show, however, a somewhat lower declination for the

sun's apex than the brighter ones. Since the fainter stars have been observed mainly in the Northern Hemisphere, the declination of the apex is somewhat uncertain.

7. The preferential motions of three different groups of stars have been studied, the groups including stars of different luminosities and distances. A general expression for a surface representing a continuous variation of the radius vector (=average radial velocity) with direction has been used. This takes into account not only symmetrical terms affecting opposite points in the sky equally, but also asymmetrical terms with opposite effects at opposite points. The major axis of this surface agrees well with the axis of preferential motion usually adopted, if only terms of even order are used. Adding terms of odd order (asymmetrical terms), we find that the two axes of maximum radial velocity are not in a straight line, but are directed toward points near the galactic equator, and differing in galactic longitude by 134°, 143°, and 183°, respectively, for the three groups. The best approximation for a symmetrical plane perpendicular to the galaxy has for the three groups galactic longitudes 259°, 257°, and 254°, which agree well with the longitude of the center of the stellar system as determined by Charlier and by Walkey. This perhaps indicates that the stars studied are mainly moving around the center of the galactic system, with a preferential motion in the galactic plane.

MOUNT WILSON SOLAR OBSERVATORY
October 1917

# PHOTO-VISUAL MAGNITUDES OF THE STARS IN THE PLEIADES

By HARRIET McWILLIAMS PARSONS

Although many photometric workers have investigated the stars in the Pleiades both visually and photographically, a determination of photo-visual magnitudes seemed advisable. This determination furnishes (1) the data for testing the photo-visual scale by comparing photo-visual with visual magnitudes, and (2) the data for finding directly the color-indices of the stars measured by comparing photo-visual with photographic magnitudes.

The term photo-visual is self-explanatory, i.e., visual magnitudes obtained photographically are called photo-visual. To obtain such magnitudes it is necessary to use a combination of plate and color-filter that will give a spectral intensity-curve like that of the human eye, having maximum sensitiveness at $\lambda = 5500$ approximately. For the determination of photo-visual magnitudes by Professor Parkhurst of the Yerkes Observatory, Cramer Isochromatic plates, both instantaneous and medium, are used, combined with carefully prepared and tested yellow color-filters known at Yerkes as $\beta$ 10 (made by R. J. Wallace) and W12 (made by C. E. K. Mees). In order to determine the spectral intensity-curve for these combinations, exposures to northern skylight were taken with a Wallace spectrograph containing Wallace grating replica No. 53. The maximum sensitiveness of the Cramer Isochromatic plate with either filter occurs at $\lambda = 5500$, although W12 cuts off more sharply in the blue than does $\beta$ 10. Eberhard has stated that it is not safe to rely upon an untested grating in a case like this, since the relative intensities in various parts of the spectrum may vary greatly with different gratings or grating replicas. An attempt was made, at the suggestion of Professor Parkhurst, to compare Wallace replica No. 53 with another Wallace replica used in the Ryerson Physical Laboratory at the University of Chicago. No noticeable difference in relative intensity was found. However, this subject

should be more thoroughly investigated, and it would be well to compare a number of gratings with a "standard prism," that is, one of which the constants are accurately determined. Judging from the results of the present investigation, this combination of color-filter and Cramer Isochromatic plates gives the desired photo-visual magnitudes.

The problem of a star's color is an important one in stellar photometry. The difference between the photographic and visual magnitudes is known as the color-index, which is a measure of the "redness" of the star. This color-index has been found to be closely related to the spectral type, i.e., the more advanced the spectral type the greater the color-index. For faint stars this relation gives a means of inferring the spectral type if the color-index is known. Also the fainter stars seem to be redder, according to present information, and it is seen that the color-indices are greater for the fainter stars.

Of the twenty plates used in this determination, fifteen were taken with the Zeiss doublet camera by Professor Parkhurst and the writer, and five with the Yerkes two-foot reflector. The doublet has lenses of "ultra-violet" glass and is of the Petzval type, with aperture of 14.5 cm and focal length of 81.4 cm. This instrument has a two-inch guiding telescope and gives sharp focal images. One of two parallel-wire gratings, known as R8 and R9, is placed over the objective for part of the exposure. In the case of the former the diameter of the wire is equal to the free space. Only two plates used here were taken with this grating. Since we wish to compare "free" images (those taken without the grating) with the central and first diffraction images obtained with the grating, it is necessary that atmospheric conditions should remain constant during the two exposures. If the sky is changing uniformly, it is still possible to make the comparison by using the mean of the first and third exposures, taken with the grating, compared with the second exposure, taken without the grating. The plates taken with the two-foot reflector were useful in determining faint magnitudes. Since the field of the reflector is small, only a part of the region of the Pleiades may be photographed on a plate. The images are not so sharp or so distinct as those obtained with the camera, because

guiding is more difficult. The grating used with this instrument is known as AI.

The engraving, Plate IV, was made from two separate prints, the junction being faintly visible east of Alcyone, between the stars 241 and 547. The two parts are separated by 2 mm too much space. The scale of the eastern part is 1 per cent greater than that of the western, which is 1 mm = 35.″5.

The plates were measured with the Hartmann "Mikrophotometer." The measurement consists in comparing the stellar images with an artificial scale formed by successive exposures on a given star, varying in a constant ratio. After the plate is measured, a correction is applied to reduce to the center of the plate. This correction is necessary because the size of an image varies slightly with its distance from the optical axis. On the camera plates the correction is small, while on the reflector plates it is much larger and more uncertain.

The next step in the reduction consists in changing the corrected scale-readings into relative magnitudes. This is accomplished by means of the known intervals of magnitude obtained by the use of the grating. With a given grating the difference in magnitude between the "free" and central image from grating is

$$5 \log_{10} \frac{a+d}{a}. \qquad (1)$$

The interval in magnitude between the central and first diffraction image is given by

$$5\left(\log_{10} \frac{a\pi}{a+d} - \log \sin \frac{a\pi}{a+d}\right), \qquad (2)$$

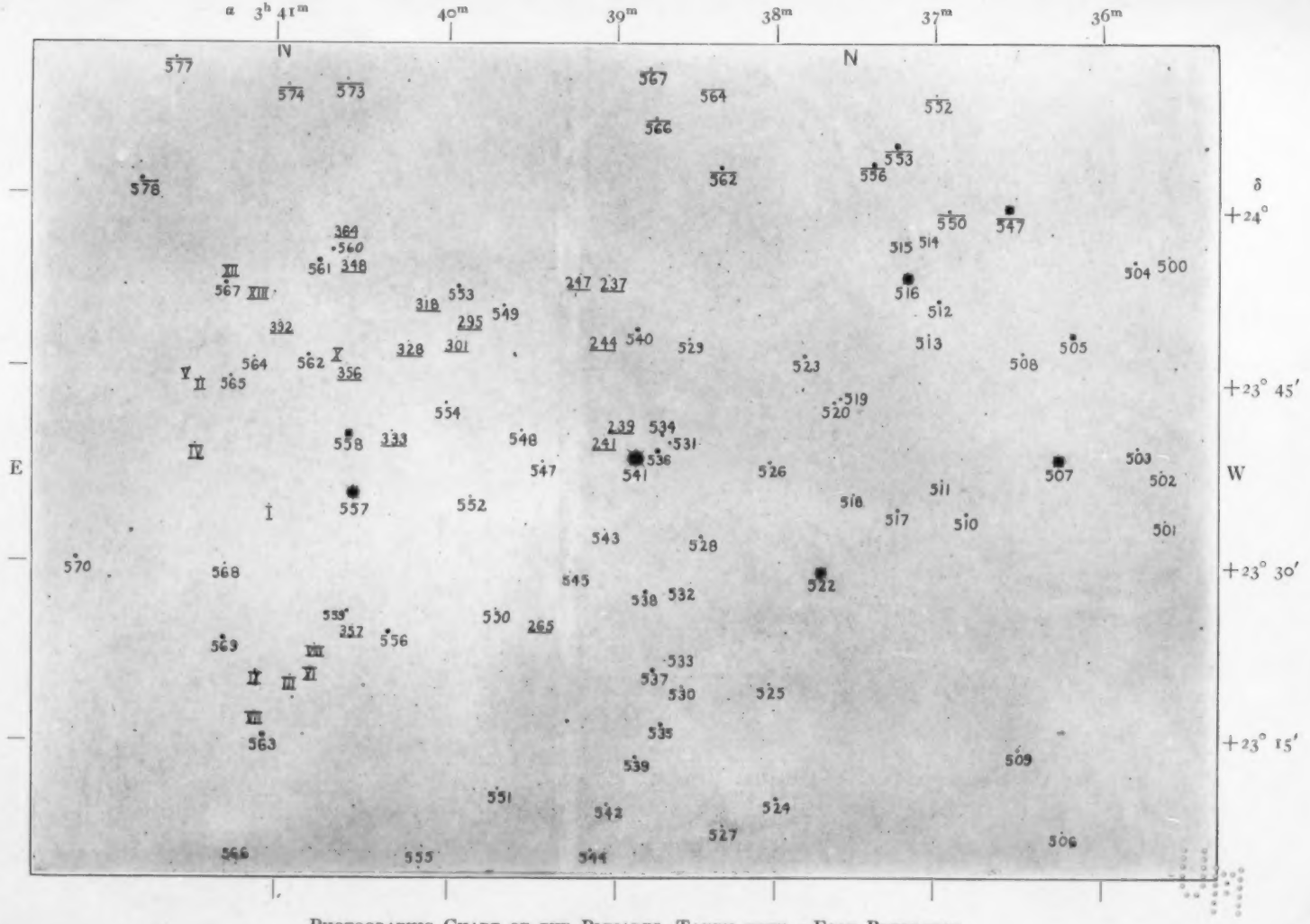
where $a$ = free space between wires, $d$ = diameter of wires.[1] The method used in reducing is substantially that used by Miss Leavitt and explained in *Harvard Annals*, **71**, 147.

The last step consists in changing the relative magnitudes into magnitudes on the International Scale. Up to this point the

[1] The constants for reduction of the two gratings are:

| | R8 | R9 |
|---|---|---|
| Central − Free | $= 1^{M}.50$ | $0^{M}.68$ |
| Spectral − Central | $= 0.98$ | $2.24$ |

## PLATE IV




PHOTOGRAPHIC CHART OF THE PLEIADES, TAKEN WITH 2-FOOT REFLECTOR

The numerals are the B.D. numbers, except those underscored, which are Wolf's

reduction has been independent of other determinations, depending only upon the constants of the grating. Stars of spectral type A between magnitudes 5.5 and 6.5 are called standard stars, since their photographic and visual magnitudes are assumed to be equal. Seven of these standards[1] were used in this work, the determinations of magnitude by Müller and Kempf[2] being taken as a basis, reduced to the International Scale by subtracting $0^{M}.30$.

In the accompanying tables are found the photo-visual magnitudes of 111 stars in the Pleiades, the visual magnitudes according to Müller and Kempf, the photographic magnitudes determined by Münch from the plates taken by Hertzsprung, the differences between the visual and photo-visual magnitudes, and the color-index. The second table gives the fainter stars which have Wolf's designation (underscored on the engravings), and thirteen stars with Roman numerals for identification upon the photographs. The third table is in the form of a summary, for classes of stars arranged according to magnitude, giving the average deviations from the mean for the different plates used (showing accordance of the photo-visual magnitudes), the means of the differences between visual and photo-visual magnitudes, and the means of the color-indices. Table IV classifies the stars according to color-index, giving the means of the color-indices and the means of the differences between visual and photo-visual magnitudes.

Tables III and IV give the two important results of the investigation. In Table III the column of mean differences between visual and photo-visual magnitudes shows whether our photo-visual scale differs systematically from that of Müller and Kempf, and whether the color-index changes with the brightness. In regard to the first point, a slight increase is noted as the magnitudes increase, which may be due to the fact that the determinations of magnitude for the fainter stars are more uncertain, since photographic images are often indistinct. The color-index is seen to increase with the fainter stars. Rosenberg[3] finds an increase in the redness of the stars in the Pleiades as they become fainter, especially

[1] B.D.+23° 505, +22 563, +24 546, +24 553, +23 563, +23 536, +24 556.

[2] *Astronomische Nachrichten*, **150**, 193, 1899.

[3] *Ibid.*, **186**, 71, 1910.

TABLE I

| B.D. No. | Photo-Visual Mag. | Müller and Kempf Visual Mag. | Münch Photog. Mag. | M & K−Ps | Color-Index |
|---|---|---|---|---|---|
| +23° 541 | 2.94 | 2.89 | 2.88 | −0.05 | −0.06 |
| 23 557 | 3.73 | 3.62 | 3.60 | −0.11 | −0.13 |
| 23 507 | 3.76 | 3.66 | 3.59 | −0.10 | −0.17 |
| 23 516 | 3.93 | 3.91 | 3.80 | −0.02 | −0.13 |
| 23 522 | 4.28 | 4.18 | 4.08 | −0.10 | −0.20 |
| +24 547 | 4.34 | 4.27 | 4.17 | −0.07 | −0.17 |
| 23 558 | 5.15 | 5.08 | 4.96 | −0.07 | −0.19 |
| 23 505 | 5.53 | 5.52 | 5.35 | −0.01 | −0.18 |
| 22 563 | 5.59 | 5.54 | 5.36 | −0.05 | −0.23 |
| 24 546 | 5.73 | 5.68 | 5.52 | −0.05 | −0.21 |
| +24 553 | 5.83 | 5.87 | 5.68 | +0.02 | −0.17 |
| 23 563 | 6.23 | 6.21 | 6.07 | −0.02 | −0.16 |
| 23 536 | 6.24 | 6.38 | 6.35 | +0.12 | +0.09 |
| 23 556* | 6.56 | 6.42 | 6.86 | −0.14 | +0.30 |
| 24 556 | 6.47 | 6.45 | 6.33 | −0.02 | −0.14 |
| +23 561 | 6.66 | 6.71 | 6.54 | +0.05 | −0.12 |
| 23 569 | 6.82 | 6.80 | 6.72 | −0.02 | −0.10 |
| 24 562 | 6.88 | 6.85 | 6.73 | −0.03 | −0.15 |
| 23 540 | 6.93 | 6.88 | 6.82 | +0.05 | −0.01 |
| 23 553 | 6.99 | 6.93 | 7.03 | −0.06 | +0.04 |
| +23 535* | 7.07 | 6.94 | 7.50 | −0.13 | +0.43 |
| 23 570 | 6.98 | 6.98 | 7.00 | 0.00 | +0.02 |
| 23 537 | 7.04 | 7.01 | 6.96 | −0.03 | −0.08 |
| 23 512 | 7.18 | 7.23 | 7.22 | +0.05 | +0.04 |
| 23 538 | 7.19 | 7.23 | 7.17 | +0.04 | −0.02 |
| +23 523 | 7.37 | 7.33 | 7.38 | −0.04 | +0.01 |
| 24 578 | 7.46 | 7.48 | 7.32 | +0.02 | −0.14 |
| 23 560 | 7.54 | 7.52 | 7.54 | −0.02 | 0.00 |
| 23 567 | 7.44 | 7.54 | 7.55 | +0.10 | +0.11 |
| 23 519* | 7.69 | 7.54 | 9.06 | −0.15 | +1.37 |
| +24 566 | 7.68 | 7.69 | 7.82 | +0.01 | +0.14 |
| 23 542* | 7.82 | 7.73 | 9.30 | −0.09 | +1.48 |
| 23 539 | 7.81 | 7.75 | 7.82 | −0.06 | +0.01 |
| 23 517 | 7.87 | 7.82 | 7.95 | −0.05 | +0.08 |
| 23 520 | 7.94 | 7.99 | 8.21 | +0.05 | +0.27 |
| +23 562 | 8.01 | 8.01 | 8.05 | 0.00 | +0.04 |
| 23 510 | 8.06 | 8.03 | 8.15 | −0.03 | +0.09 |
| 23 524 | 8.13 | 8.10 | 8.41 | −0.03 | +0.28 |
| 23 559 | 8.14 | 8.13 | 8.31 | −0.01 | +0.17 |
| 23 504 | 8.22 | 8.14 | 8.29 | −0.08 | +0.07 |
| +23 549 | 8.31 | 8.23 | 8.52 | −0.08 | +0.21 |
| 24 563 | 8.33 | 8.24 | 8.61 | −0.09 | +0.28 |
| 23 549 | 8.27 | 8.32 | 8.43 | +0.05 | +0.16 |
| 23 528 | 8.46 | 8.35 | 8.51 | −0.11 | +0.05 |
| 24 550* | 8.69 | 8.55 | 8.78 | −0.14 | +0.09 |

TABLE I—*Continued*

| B.D. No. | Photo-Visual Mag. | Müller and Kempf Visual Mag. | Münch Photog. Mag. | M & K−Ps | Color-Index |
|---|---|---|---|---|---|
| +23°508 | $8^{M}51$ | $8^{M}66$ | $8^{M}78$ | $+0^{M}15$ | $+0^{M}27$ |
| 23 544* | 8.88 | 8.70 | 10.15 | −0.18 | +1.27 |
| 23 531 | 8.68 | 8.72 | 9.05 | +0.04 | +0.37 |
| 24 567* | 8.91 | 8.75 | 8.76 | −0.16 | −0.15 |
| 23 503 | 9.12 | 8.99 | 9.34 | −0.13 | +0.22 |
| +23 509 | 8.90 | 9.04 | 9.20 | +0.14 | +0.30 |
| 22 570 | ..... | 9.10 | ..... | ..... | ..... |
| 23 554 | 9.12 | 9.11 | 9.34 | −0.01 | +0.22 |
| 23 565* | 9.14 | 9.14 | 9.19 | 0.00 | +0.05 |
| 23 530* | 9.37 | 9.27 | 9.75 | −0.10 | +0.38 |
| +23 548 | 9.32 | 9.28 | 9.61 | −0.04 | +0.29 |
| 24 577 | 9.20 | 9.33 | 9.51 | +0.13 | +0.31 |
| 23 526 | 9.30 | 9.35 | 9.55 | +0.05 | +0.25 |
| 23 564* | 9.46 | 9.37 | ..... | −0.09 | ..... |
| 23 513 | 9.47 | 9.40 | 9.73 | −0.07 | +0.26 |
| +23 529 | 9.47 | 9.46 | 9.71 | −0.01 | +0.24 |
| 22 568 | ..... | 9.53 | ..... | ..... | ..... |
| 23 506 | 9.10 | 9.58 | ..... | +0.48 | ..... |
| 24 552* | 9.40 | 9.70 | 10.10 | +0.30 | +0.70 |
| 24 545 | 9.71 | 9.73 | 10.18 | +0.02 | +0.47 |
| +23 545 | 9.53 | 9.76 | 10.21 | +0.23 | +0.68 |
| 23 568 | 9.56 | 9.78 | 10.19 | +0.22 | +0.63 |
| 24 574 | 9.52 | 9.84 | ..... | +0.32 | ..... |
| 23 566 | 9.49 | 9.85 | ..... | +0.36 | ..... |
| 23 546 | ..... | 9.90 | ..... | ..... | ..... |
| +23 547 | 9.58 | 9.90 | 10.26 | +0.32 | +0.68 |
| 23 552 | 9.82 | 9.91 | 10.39 | +0.09 | +0.57 |
| 23 551 | 9.82 | 9.99 | 10.50 | +0.17 | +0.68 |
| 22 560 | 10.10 | 10.01 | ..... | −0.09 | ..... |
| 23 525 | 10.43 | 10.05 | 10.61 | −0.38 | +0.18 |
| +23 511 | 10.25 | 10.06 | 10.48 | −0.19 | +0.23 |
| 24 564 | ..... | 10.12 | ..... | ..... | ..... |
| 24 579 | ..... | 10.13 | ..... | ..... | ..... |
| 23 550 | 9.83 | 10.22 | 10.72 | +0.39 | +0.89 |
| 24 573 | ..... | 10.23 | 10.94 | ..... | ..... |
| +23 500 | ..... | 10.24 | ..... | ..... | ..... |
| 23 555 | 9.48 | 10.31 | ..... | +0.83 | ..... |
| 23 502 | ..... | 10.41 | ..... | ..... | ..... |
| 23 527 | ..... | 10.42 | ..... | ..... | ..... |
| 23 533* | 10.70 | 10.42 | ..... | −0.28 | ..... |
| +23 518 | 10.45 | 10.43 | ..... | −0.02 | ..... |
| 23 532 | 10.53 | 10.44 | ..... | −0.09 | ..... |
| 23 521 | ..... | 10.46 | ..... | ..... | ..... |
| 24 541 | ..... | 10.52 | ..... | ..... | ..... |
| 23 543 | 10.23 | 10.54 | ..... | +0.32 | ..... |
| +23 501 | ..... | 10.78 | ..... | ..... | ..... |
| 23 515 | ..... | 10.78 | ..... | ..... | ..... |
| 24 542 | ..... | 10.75 | ..... | ..... | ..... |
| 23 514 | ..... | 11.00 | ..... | ..... | ..... |
| +24 544 | ..... | 11.51 | ..... | ..... | ..... |

TABLE II*

| Wolf's No. | Ps No. | Photo-Visual Mag. | Wolf's No. | Ps No. | Photo-Visual Mag. |
|---|---|---|---|---|---|
| 318 | | $9^{M}.98$ | | VII | $10^{M}.87$ |
| | I | 10.01 | | VIII | 10.91 |
| | II | 10.09 | | IX | 10.92 |
| 247 | | 10.18 | 364 | | 10.95 |
| 328 | | 10.19 | 356 | | 11.02 |
| | III | 10.29 | 341 | | 11.02 |
| 265 | | 10.30 | | X | 11.03 |
| | IV | 10.31 | 348 | | 11.07 |
| 335 | | 10.33 | | XI | 11.07 |
| | V | 10.36 | 289 | | 11.09 |
| 301 | | 10.40 | 244 | | 11.10 |
| 333 | | 10.44 | 267 | | 11.10 |
| | VI | 10.64 | 295 | | 11.14 |
| 239 | | 10.74 | 357 | | 11.17 |
| 241 | | 10.82 | | XII | 11.27 |
| | | | | XIII | 11.29 |

In these tables Ps (Parsons) refers to the values of the present paper.

TABLE III

| Mag. (M & K) | Average Deviation from Mean | Mean Difference M & K−Ps | | No. of Stars | Mean Color-Index | No. of Stars |
|---|---|---|---|---|---|---|
| | | Arithmetical | Algebraic | | | |
| $3^{M}.0-5^{M}.5$... | $\pm 0^{M}.13$ | $-0^{M}.07$ | $-0^{M}.07$ | 7 | $-0^{M}.15$ | 7 |
| 5.5− 6.5... | ±0.06 | −0.05 | −0.02 | 8 | −0.09 | 8 |
| 6.5− 7.5... | ±0.10 | ±0.04 | −0.01 | 12 | −0.01 | 12 |
| 7.5− 8.5... | ±0.13 | ±0.06 | −0.03 | 17 | +0.26 | 17 |
| 8.5− 9.5... | ±0.19 | ±0.09 | −0.03 | 16 | +0.28 | 15 |
| 9.5−10.5... | ±0.18 | ±0.27 | +0.15 | 18 | +0.57 | 10 |
| 10.5−11.5... | ..... | +0.32 | +0.32 | 1 | ..... | ..... |

TABLE III*a*

| Mag. (M & K) | Average Deviation from Mean | Mean Difference M & K−Ps | | No. of Stars | Color-Index |
|---|---|---|---|---|---|
| | | Arithmetical | Algebraic | | |
| $3^{M}.0-5^{M}.5$..... | $\pm 0^{M}.13$ | $-0^{M}.07$ | $-0^{M}.07$ | 7 | $-0^{M}.15$ |
| 5.5− 6.5..... | ±0.06 | ±0.04 | −0.01 | 7 | −0.12 |
| 6.5− 7.5..... | ±0.16 | ±0.04 | −0.01 | 11 | −0.04 |
| 7.5− 8.5..... | ±0.13 | ±0.05 | −0.04 | 14 | +0.13 |
| 8.5− 9.5..... | ±0.19 | ±0.09 | −0.03 | 12 | +0.21 |
| 9.5−10.5..... | ±0.18 | ±0.27 | +0.14 | 17 | +0.57 |
| 10.5−11.5..... | ..... | +0.32 | +0.32 | 1 | ..... |

between the seventh and ninth magnitudes, which change in color he considers a real function of the "Glühstand," and not due to the absorption in space. Hertzsprung[1] states that the color-index from sixth to ninth magnitude increases to $0^{M}.3$, which fact may be noted in Table III. Table IV shows whether the combination of plate and color-filter differs in color-perception from the Müller and Kempf eye. From the results any differences might seem to be accidental.

TABLE IV

| Color-Index | No. of Stars | Mean Color-Index | Mean Difference M & K−Ps | |
|---|---|---|---|---|
| | | | Arithmetical | Algebraic |
| $-0^{M}.25-0^{M}.00$ | 22 | $-0^{M}.13$ | $\pm 0^{M}.05$ | $-0^{M}.03$ |
| 0.00−+0.20 | 18 | +0.08 | ±0.07 | −0.04 |
| +0.20−+0.50 | 19 | +0.29 | ±0.08 | −0.03 |
| +0.50−+1.00 | 7 | +0.69 | +0.25 | +0.25 |
| +1.00−+1.50 | 3 | +1.37 | −0.14 | −0.14 |

Trümpler[2] states that ten stars in the Pleiades by their proper motions are shown to have no physical connection with the rest of the group. Omitting these ten stars (having asterisks in Table I), we have Table III*a*, similar to III. On the whole, the results would seem to indicate that the scale employed is practically the same as that of Müller and Kempf, and that the combination of plate and color-filter has color-perception similar to that of the human eye.

Whatever of merit may be found in these determinations of magnitude and the results deduced is due to the suggestion and guidance of Professor J. A. Parkhurst, of the Yerkes Observatory, who took several of the plates used and under whose direction the other plates were taken, measures made, computation carried out, and the subject discussed in its present form. To Professor Parkhurst I wish to express my gratitude and appreciation.

Vassar College Observatory
January 1917

[1] *Astronomische Nachrichten*, **200**, 137, 1915. [2] *Ibid.*, **200**, 222, 1915.

# NOTE ON THE CEPHEID VARIABLE SU CASSIOPEIAE[1]

By WALTER S. ADAMS and HARLOW SHAPLEY

A variation in the radial velocity of Boss 637, SU Cassiopeiae,[2] was found by Adams in the course of the regular spectroscopic work with the 60-inch reflector. The observations and measures were made without knowledge of the variations in light, as the star is not listed as variable in the *Preliminary General Catalogue*. The spectrograms on hand permit an inquiry into spectroscopic parallax, variation of velocity, and changes in spectral type, and the last may be compared with the analogous variations previously found from objective-prism plates. The material is discussed in the present note, which also includes some remarks on the variation in light.

The variability in brightness was detected in 1906 by Müller and Kempf[3] from the discordances in measures for the *Potsdam Photometric Durchmusterung*. A special photometric study indicated a variation between magnitudes 5.93 and 6.26, with a period defined by the formula,

$$\text{Max.} = \text{J.D. } 2417287.30 + 1.9498\,E, \quad \text{G.M.T.}$$

The visual light-curve was found to differ from that of typical Cepheids in its small amplitude and symmetrical form.

In 1906–1908 the star was observed photographically by J. A. Parkhurst,[4] who also found the variation peculiarly small and nearly symmetrical with respect to maximum light. The elements by Müller and Kempf being unsatisfactory, the period $1^d.9490$ was substituted for the value above. From the photographic magnitudes at maximum and minimum, 6.52 and 6.99, respectively, Parkhurst derived a variation in color-index from 0.59 to 0.73. An objective-prism plate at phase $1^d.28$ gave a spectrum of F3, in

[1] *Contribution from the Mount Wilson Solar Observatory*, No. 145.

[2] $\alpha = 2^h43^m2^s.9$, $\delta = +68°28'27''$ (1900.0).

[3] *Astronomische Nachrichten*, **173**, 307, 1907.

[4] *Astrophysical Journal*, **28**, 279, 1908.

fair agreement with the color-index. Because of the star's faintness, the attempt to find a variation in radial velocity with the Bruce spectrograph and the 40-inch refractor was considered inconclusive, although the observed range was from −1 km to −16 km.

An examination of the light-curve by Shapley[1] has shown that the variation cannot be interpreted as the result of the rotation of a simple ellipsoidal body. The conclusion that the star is a Cepheid, notwithstanding the form and amplitude of its light-curve, is verified by the spectroscopic work described below.

Variations in spectral type from A8 to F5 were recorded in 1916 on plates made with the 10-inch photographic telescope at Mount Wilson.[2] The type of spectrum and the brightness of the spectral images (compared with those of neighboring stars) indicated that neither the formula by Müller and Kempf nor its modification by Parkhurst then represented the variation in light; but the epoch given by Müller and Kempf with an intermediate value of the period, $1^d.94935$, was found sufficiently accurate for the work on spectral variation.

A new investigation of the period has now been made, employing (1) the observations made at the Potsdam and Yerkes observatories; (2) a redetermination of the magnitudes on the objective-prism plates of 1915 and 1916, and (3) a series of visual estimates on four nights in August 1917. The result has been checked with the observations of spectral type mentioned in the preceding paragraph, as well as those derived from the slit spectrograms, and with the observations of radial velocity given below. The variations in light, velocity, and spectrum are all relatively small and gradual, making high accuracy for the elements impossible. The following formula, however, gives satisfactory phases for all variations:

$$\text{Max.} = \text{J.D. } 2417287.10 + 1.9495\,E, \quad \text{G.M.T.}$$

There is some evidence that the period is not constant.

The spectroscopic observations with the 60-inch telescope are given in Table I. The phases have been computed from the

[1] *Astronomische Nachrichten*, **194**, 357, 1913.

[2] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

preceding maximum of the light-curve. Describing the velocity-variations in terms of the orbital elements of a spectroscopic binary, the following results are obtained from a graphical solution:

$$\text{Epoch of Max. Vel. of Approach} = \text{J.D. } 2420197.65, \text{ G.M.T.}$$
$$\text{Period} = 1^d.9495$$
$$e = 0.0$$
$$K = 11.0 \text{ km/sec.}$$
$$V_0 = -7.0 \text{ km/sec.}$$
$$a \sin i = 295{,}000 \text{ km}$$
$$\frac{m^3 \sin^3 i}{(m+m_1)^2} = 0.0003 .$$

The epoch of maximum negative velocity precedes the maximum of light by $0^d.05$. The elements are entirely typical of Cepheid

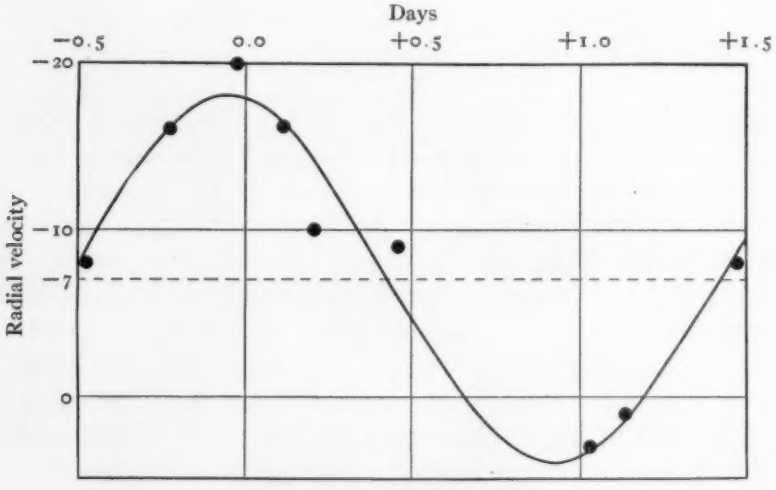



FIG 1.—Velocity-curve of SU Cassiopeiae

variables that have small ranges of velocity and nearly symmetrical velocity-curves. In Fig. 1 the observations are plotted along the computed velocity-curves. The average deviation is ±1.5 km.

The spectral types determined by Adams from the spectrograms made with the 60-inch reflector are given in the last column of

Table I. The types derived by Shapley from the plates made with the 10-inch refractor are in Table II,[1] with phases based on

TABLE I

OBSERVATIONS WITH 60-INCH REFLECTOR

| Plate | Date | J.D. and G.M.T. | Epoch | Phase | Velocity | Spectrum |
|---|---|---|---|---|---|---|
| | | | | | km | |
| γ 2897.... | 1913 Nov. 16 | 2420088.745 | 1437 | $0^d.21$ | −10 | F2 |
| 3072.... | 1914 Feb. 1 | 0165.692 | 1476 | 1.13 | + 1 | F5 |
| 3171.... | Mar. 5 | 0197.675 | 1492 | 1.93 | −20 | Fo |
| 3898.... | Nov. 24 | 0461.915 | 1628 | 1.04 | + 3 | F6 |
| 5116.... | 1916 Oct. 7 | 1144.943 | 1978 | 1.73 | −16 | Fo |
| 5239.... | Nov. 9 | 1177.821 | 1995 | 1.47 | − 8: | F3 |
| 5311.... | Dec. 7 | 1205.706 | 2010 | 0.12 | −16 | F2 |
| 6200.... | 1917 Sept. 6 | 1478.986 | 2150 | 0.47 | − 9 | F4 |

the new elements. The variation of spectrum with magnitude is illustrated in Fig. 2.

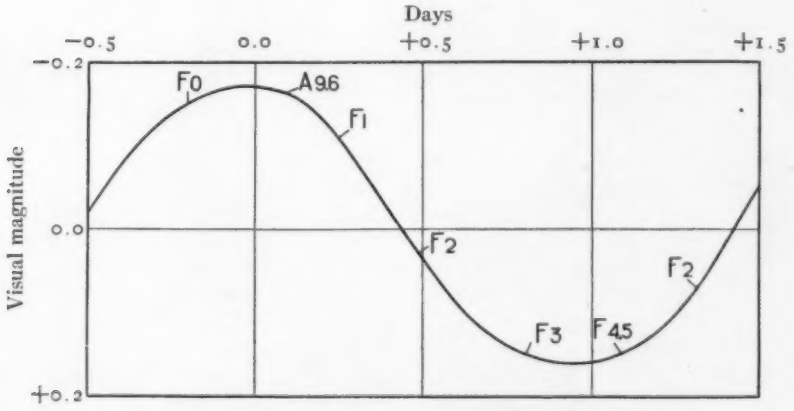



FIG. 2.—Variation of the spectral type of SU Cassiopeiae

The proper motion of SU Cassiopeiae is mainly parallactic, being 0".016 in position-angle 108°. The parallax, according to a

[1] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916. Types marked as uncertain in the earlier paper are now omitted, and a typographical error is corrected. A new classification of all plates yielded but one correction; for the plate on Julian Day 2420841 the spectrum is F3 instead of F1.

recent determination by van Maanen,[1] is +0".010 ±0.003; the spectroscopic parallax is in exact agreement if the visual magnitude

TABLE II

OBSERVATIONS WITH 10-INCH REFRACTOR

| J.D. and G.M.T. | Epoch | Phase | Spectrum | J.D. and G.M.T. | Epoch | Phase | Spectrum |
|---|---|---|---|---|---|---|---|
| 2420841.836... | 1823 | 0d80 | F3 | 2420900.727... | 1853 | 1d21 | F2 |
| 0894.775... | 1850 | 1.11 | F3 | 0900.731... | " | 1.21 | F1 |
| 0894.782... | " | 1.11 | F4 | 0901.685... | 1854 | 0.21 | A9 |
| 0896.711... | 1851 | 1.09 | F5 | 0901.694... | " | 0.22 | Fo |
| 0899.622... | 1853 | 0.10 | A9 | 0901.702... | " | 0.23 | F2 |
| 0899.631... | " | 0.11 | A8 | 0901.714... | " | 0.24 | F1 |
| 0899.820... | " | 0.30 | F2 | 0901.725... | " | 0.25 | Fo |
| 0899.832... | " | 0.31 | Fo | 0901.737... | " | 0.27 | Fo |
| 0900.722... | " | 1.20 | F2 | | | | |

is taken to be 6".23, as given in the *Preliminary General Catalogue*. With the corresponding value of the distance, the total velocity in space with respect to the sun is 10 kilometers a second.

MOUNT WILSON SOLAR OBSERVATORY
November 1917

[1] *Mt. Wilson Contr.*, No. 136, 1917.

# *Minor Contributions and Notes*

## STARS IN THE GROUP OF THE PLEIADES BUT NOT BELONGING TO THE PHYSICAL SYSTEM

In another place in this number is printed Miss Parsons' paper entitled "Photo-visual Magnitudes of Stars in the Pleiades," in which she calls attention to Trümpler's list[1] of twelve stars projected on the group but having proper motions so different from the rest as to suggest detachment from the system. Most of these stars possess the added peculiarity of a larger color-index than the rest of the group. The accompanying table, which collects the data from the two papers cited, will therefore be of interest.

TABLE I

| B.D. | Magnitudes | | Color-Index | | Relative Proper Motion |
|---|---|---|---|---|---|
| | Photo-visual Parsons | Photographic Muench | Parsons | Trümpler | |
| +23°519..... | $7^{M}69$ | $9^{M}06$ | $+1^{M}37$ | $+1^{M}39$ | 2".51 |
| 530..... | 9.37 | 9.75 | +0.38 | +0.40 | 5".25 |
| 533..... | 10.70 | (11.3) | (+0.6) | ..... | (5".63) |
| 535..... | 7.07 | 7.50 | +0.43 | +0.53 | 5".48 |
| 542..... | 7.82 | 9.30 | +1.48 | +1.23 | 3".39 |
| +23°544..... | 8.88 | 10.15 | +1.27 | +1.17 | 3".86 |
| 556..... | 6.56 | 6.86 | +0.30 | +0.40 | 2".40 |
| 564..... | 9.46 | (10.6) | (+1.1) | (+2m) | 5".18 |
| 565..... | 9.14 | 9.19 | +0.05 | −0.15 | 4".17 |
| +24°550..... | 8.69 | 8.78 | +0.09 | +0.13 | 2".12 |
| 552..... | (9.40) | 10.10 | (+0.70) | +0.25 | 2".11 |
| 567..... | 8.91 | 8.76 | −0.15 | −0.13 | 3".70 |
| Mean..... | ..... | ..... | +0.76 | +0.75 | ..... |

In the table the magnitudes and color-indices are given from Miss Parsons' paper in columns two, three, and four, with the following additions:

1. B.D. +23°533: The photographic magnitude of this star is not given explicitly by Muench, but comparisons with neighboring

[1] *Astronomische Nachrichten*, **200**, 222, 1915.

stars on his photographic print show the value to be about $11^{M}.3$, with a resulting color-index of +0.6.

2. B.D. $+23^{\circ}564$: By the same process as used for the foregoing star the magnitude 10.6 and the color-index +1.1 were obtained.

3. B.D. $+24^{\circ}552$: This is the only star for which the color-index found by Miss Parsons differs materially from Trümpler's value. Investigation of the original data showed that it had been measured by Miss Parsons on only one plate and that the image was somewhat elongated, so that the resulting magnitude could not be reliable and the star should be omitted from the list.

4. Omitting the above-mentioned data, inclosed in parentheses, the mean of the color-indices of the remaining nine stars is almost identical in the two lists, differing by only 1 per cent.

From the foregoing data and the list in Miss Parsons' paper we therefore reach the following remarkable conclusions:

1. Down to magnitude 8.7 all stars having color-indices as large as 0.30 are shown by their large relative proper motions (compared with Alcyone) to belong outside the system.

2. All stars (with two or perhaps three exceptions) having large relative proper motions have also large color-indices and are therefore in a different stage of evolution from the members of the system.

Thus we have some additional data tending to prove the homogeneity of the physical system of the Pleiades.

J. A. PARKHURST

YERKES OBSERVATORY
January 1918

## CHANGES IN THE SPECTRUM OF THE WOLF-RAYET STAR GAMMA ARGUS

Spectrographic investigations were recently begun of bright-line hydrogen stars, the Wolf-Rayet stars, and the brighter stars of Classes B and A. Among the first of the Wolf-Rayet stars to be observed was the premier star of this class, $\gamma$ Argus. These observations show unmistakable changes in its spectrum. On account of the importance of the star a preliminary note on the subject is thought desirable.

The observations so far made have been obtained with a 20° prism of 5 inches aperture attached to the astrographic equatorial as an objective-prism. This combination gives spectra in good focus for practically the entire photographic region, the dispersion at Hγ being 35 A per millimeter. This dispersion is sufficient to show clearly the widening of lines in spectra of early types and much detail. The resolution is good, the H and Hε lines in Sirius, for example, being clearly separated. The negatives were made on Seed 30 plates.

Several exposures were secured on August 15, 16, and 17 and a fourth plate with three different exposures on November 13, 1917. In this interval of three months there has been a well-defined change in the structure of Hβ and a less marked change in the region of λ 450. At both epochs there is weak, narrow Hβ absorption, with bright borders of unequal intensity on the two sides. In August the bright border on the red side is the stronger. In November the violet border is the stronger. In both cases the differences are very pronounced, and their reality is confirmed by several images. I have spoken of the Hβ as an absorption-effect. It is doubtful, however, if the position of that line is in reality any weaker than the continuous spectrum outside of the bright borders, although it shows as a weak absorption line between the two brightenings.

The changes in the region of λ 450 are not so pronounced, although they appear to be certain. They are in the nature of a change of width of the bands.

A comparison of these recent observations with the Harvard reproduction of the spectrum of this star[1] and with Campbell's observations in 1893–1894[2] shows that large changes have almost certainly occurred in the interval of twenty years. Indeed, the intensities of the bands in the recent observations bear little resemblance to the Harvard reproduction. On account, however, of the well-known peculiarities which are introduced in photographic copies, it is not possible at this time to discuss the observed differences.

[1] *Harvard Annals*, **56**, No. 6, Plate II, Fig. 4.

[2] *Astronomy and Astrophysics*, **12**, 555, 1893; **13**, 457, 1894.

Campbell's statement, "At F the spectrum appears to be strictly continuous," with regard to his 1893 observation, and the statement in his later article—"Two negatives show what seems to be a very faint band at H$\beta$, with a fine, partially dark line through its center at 4861; but the contrasts are very slight indeed"—seem to furnish a secure basis for the conclusion that the structure recently observed in that region has become more pronounced. His observations were also photographic, and H$\beta$ is now much stronger than the line at $\lambda$ 479, which he also recorded.

The recent observations also show a broad, very faint brightening in the general region of the chief nebular line. Although very faint, this strengthening seems to be certain. A general strengthening is *suspected* in the region of the second nebular line.

A more detailed discussion of the spectrum of this star as well as of others will be published later.

Two conclusions are suggested by these observations: that there has been a secular change in the spectrum and that there have been changes in the structure of the H$\beta$ line analogous to those observed in the bright-line hydrogen stars. This latter conclusion appears to be especially significant as tending still further to link together the stars of Classes B and O.

I have had the efficient assistance of Second Astronomer Winter in taking the photographs.

C. D. PERRINE

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
November 14, 1917

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